# THEORY

OF

# ARCHES.

BY

Prof. W. ALLAN,

*Formerly of Washington and Lee University, Lexington, Va.*

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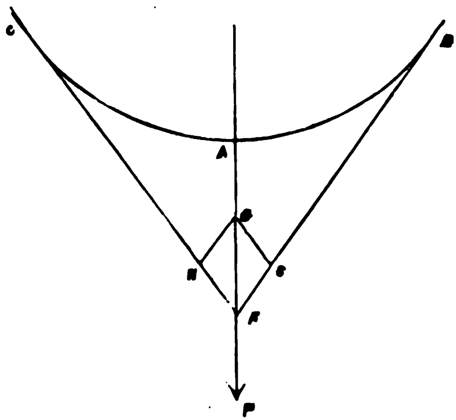
# THEORY OF ARCHES.

The following is an amplification and explanation of Professor Rankine's chapters on this subject.

Perhaps the clearest way of developing the "Theory of Arches" is to begin with the consideration of the forces which act upon a suspended chain or cord. The force in the chain or cord is just the opposite of that upon an arch—that is, it is *tension* instead of *compression*, but the relations between the "external" and "internal" forces, or what is the same, between the loads and the resistances they produce, are strictly analogous.

Let C A B (Fig. 1) be a cord suspended at C and B and loaded in any manner over its whole length. Consider the forces acting on this cord. Suppose it attached to a hook at B and to another at C. A cord without stiffness cannot exert a pull except in the direction of its length : therefore the "pulls" in the rope at C and B, and exerted at these points on the suspending hooks, must be in the direction of the tangents at those points. The load is supposed to be

Fig. 1.




distributed over the cord, but we may find its resultant. Let P be this resultant and P F its direction. The *three* forces, viz. : the pulls at C and B, and the resultant of the load, P, are all in the same vertical plane; they are the only forces acting on the cord; and as they are in equilibrium, the *directions of these three forces must meet in one point*, and the forces themselves *must be proportional to the three sides of a triangle drawn parallel to their directions.*

G N F (Fig. 1) is such a triangle. The known directions of the pulls at B and C, and of P, give us the angles in this triangle; and if we know also the magnitude of the load P, represented by the line G F, we can determine that of the pulls at B and C. For

(Pull at B = G N) : G F : : sin G F N : sin GNF.
(Pull at C = N F) . G F : : sin N G F : sin G N F.

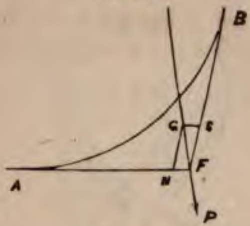
The analysis we have made for the whole cord may be applied to any part of it. Thus, if we consider any arc B′ A′ (Fig. 2) of the cord, and the load on that arc, we have three forces in the same plane in equilibrium. For at A′ and B′ the other parts of the cord may be replaced by two hooks, and the pulls on these hooks, exerted by the cord at A′ and B′, will be, as before, in the direction of the tangents at those points. The resultant P′ of the load on A′ B′ must pass through the point of intersection of the tangents, and if the direction of that resultant be as indicated in the figure, then G′ N′ F′ will be the triangle of forces.

Fig. 2.

The principles above explained enable us to calculate the " pulls " at all points of a loaded chain or cord, and consequently to fix *its size and strength to bear* a given load; or to determine the *amount, distribution, and direction of the load* necessary to produce assumed "pulls" in a cord of a given shape.

Thus suppose in the half of the loaded cord of (Fig. 1) we draw the tangents at A and B (as is done in Fig. 3), the resultant of the load P *must* pass through F, the point of intersection of the tangents. If the direction and amount of P be known, lay off

FIG. 3.




F G to represent it. Then, as above explained,

$$N F = \text{pull at A}$$

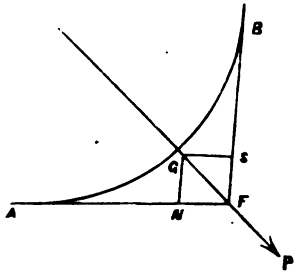
and

$$N G = \text{pull at B.}$$

Suppose, on the other hand, we assume the pulls at A and B to be equal, we lay off on the two tangents (Fig. 4), equal lengths, F S and F N, to represent these equal pulls, and upon them construct a parallelogram. Then F G gives the magnitude and direction of the resultant of the load that must be put on the cord to produce the given pulls.

A cord is in *equilibrium* when it is balanced under the load applied. Change the distribution of the load and the cord

Fig. 4.



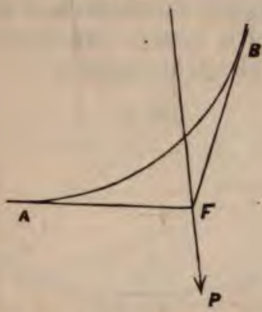

at once charges shape and assumes the form necessary to equilibrium under the new load.

Thus, if P (Fig. 5) equals the direction of the resultant of the new load on the cord from the horizontal point A to the point of support B, draw the tangent A F, until it meets the direction of the load P, at F; then draw F B. The cord A B will have so changed its form that F B (Fig. 5) will now be the direction of the tangent at B.

FORMS OF CORDS UNDER VARIOUS LOADS.

Let us now investigate the various curves which a cord will assume under different distributions of the load.

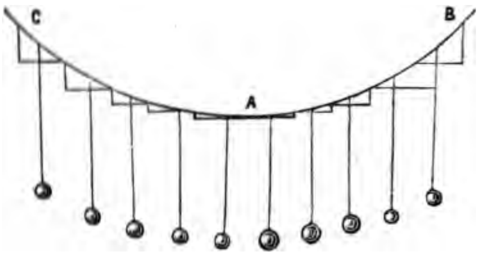
FIG. 5.




*Case I.* Suppose the load to be altogether *vertical*, and to be distributed *uniformly* along the *horizontal.*

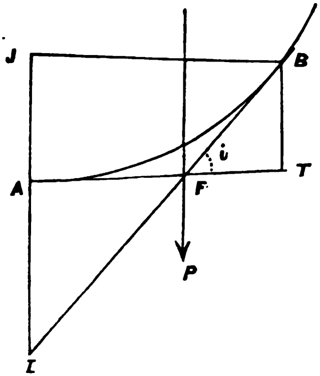
Let *equal* weights be hung, for instance, along a cord CB (Fig. 6) so that the *horizontal* distance between the threads by which the weights are suspended shall be everywhere equal. Or, draw little elementary triangles along the curve, so that the *bases* of all these little triangles shall be equal, and let the threads holding the weights cut the middle of these bases. Then each weight may be considered as the resultant of the load on the element of the curve which constitutes the hypothenuse of the little triangle to which it is attached. Such a load is *vertical* and is *uniformly distributed along the horizontal.*

FIG. 6.




To determine the curve of the cord. Obtain the resultant of the load between the horizontal point A and the point B (Fig. 7). This resultant, as the little forces are all parallel, is equal to the sum of them, and it is vertical in direction. It will also evidently bisect A T. Draw it; and from its point of intersection with A T, draw the line F B, which, as has been shown, must be tangent at B. Prolong B F to I, then the subtangent I J is seen to be bisected at the vertex A of the curve. Hence the curve C B is a *parabola*.

FIG. 7.




The triangle B F T has its sides parallel to the forces acting on the half cord A B; so that if B T be taken to represent P,

B F = pull at B
F T = pull at A.

Let T = equal tension at any point along the cord.

H = value of T at the horizontal point A, or the "horizontal pull" on the cord.

$i$ = inclination of the tangent at any point to the horizontal.

Then as the arc A F (Fig. 7) may stand for any part of the curve counting from the horizontal point A towards one of the points of suspension, we have the following general equations from the triangle B F T:

$$T^2 = P^2 + H^2 \qquad (1.)$$

$$\text{Tan } i = \frac{P}{H} = \frac{px}{H} = \frac{dy}{dx} \qquad (2.)$$

($p$ being = the load per unit of horizontal distance, A the origin of co-ordinates, A T = axis of X and A J = axis of $y$).

From equations (1) and (2) we can solve three problems.

1. Given the *curve*, and the *load*, to find T and H.

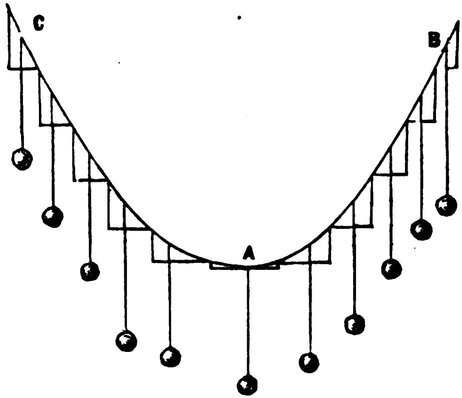
2. Given the *curve*, and T and H, to find P.

3. Given the *load*, and T and H, to find the curve.

For a full discussion of this case, see Rankine's "Civil Engineering."

Such a distribution of the load as we have discussed in the above case, is approximated to in suspension bridges, and sometimes in wood, iron, or steel arches, but not usually in stone or brick ones.

FIG. 8.




*Case II.* Let the load still be *vertical*, but distributed *uniformly along the curve.*

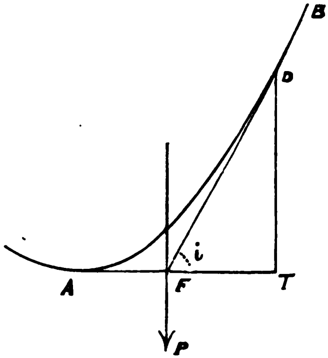
That is, divide the arc C A B (Fig. 8) into elements each of a unit in length; then the load on these elements is constant throughout. It is easily seen that such a load is not, as in the last case, uniform along the *horizontal*, for the *bases* of the little triangles of which the *hypothenuses* are now equal, diminish in extent as we go from A towards B or C. A chain of uniform material and cross-section and acted on by nothing but its own weight, is in the condition described, and, as is well known, the curve assumed by it is the "common catenary."

Let $p =$ weight of a unit's length of the cord, then if $p\,m =$ horizontal pull on the cord at A $=$ H, $m$ is called the *modulus* of the *catenary*, and represents the length of cord of the same kind as C B, the weight of which would equal the pull at A. The weight on A B $=$ P $= p\,s$ when $s =$ length of cord A B.

The triangle of forces for any arc A D (Fig. 9) can be found as before, by drawing the tangents at A and D, and the line representing the force P vertically through their intersections. The triangle D F T will represent the forces; D T being $=$ P

$= p\,s$, and F T $=$ H $= p\,m$, and D F $=$ T $=$ tension at D. Then

Fig. 9.




$$T^2 = H^2 + P^2 = p^2\,m^2 + p^2\,s^2 = p^2\,(s^2 + m^2) \quad (3.)$$

$$\text{Tan } i = \frac{D\,T}{F\,T} = \frac{p\,s}{p\,m} = \frac{s}{m} = \frac{d\,y}{d\,x} \qquad (4.)$$

From the differential equation

$$\frac{d\,v}{d\,x} = \frac{s}{m}$$

we obtain the linear equation of the curve. In doing so it is most convenient to take the origin at a point O, whose distance below the vertex A is $= m$. The line Q O X

(Fig. 10) is called the *directrix* of the catenary.

The equations of the catenary are

$$s = \frac{m}{2} \left\{ E^{\frac{x}{m}} - E^{-\frac{x}{m}} \right\} = \sqrt{y^2 + m^2} = \text{length of arc.}^* \quad (5.)$$

$$y = \frac{m}{2} \left\{ E^{\frac{x}{m}} + E^{-\frac{x}{m}} \right\} = \sqrt{s^2 + m^2} \quad (6.)$$

$$x = m.\ \text{hy. log.} \left\{ \frac{1}{\frac{y}{m} + \sqrt{\frac{y^2}{m^2} - 1}} \right\} \quad (7.)$$

$$\text{Area AOED} = \int y\,dx = m\,s \quad (8.)$$

$$\text{Tan } i = \frac{s}{m} = \tfrac{1}{2} \left\{ E^{\frac{x}{m}} - E^{-\frac{x}{m}} \right\} \quad (9.$$

$$\text{Radius of curv.} = \rho = \frac{y^2}{m^2} = \frac{m^2 + s^2}{m} \quad (10.)$$

Since the area A O E D = $m\ s$, and $m$ = a constant, the area varies as $s$. But the load on the arc A D ($= p\,s$) also varies as $s$, since $p$ is constant. Hence a convenient mode of representing the load on any arc, A D. Suppose a sheet of metal C Q T B A C (Fig. 10), bounded below by the "directrix," Q T, to be suspended from the curve. Let

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* E = Base of Naperian Logarithms.

the weight of this metal corresponding to $m$ units of its surface be $= p$. That is, let

$$w\,m = p, \text{ or } w = \frac{p}{m}.$$

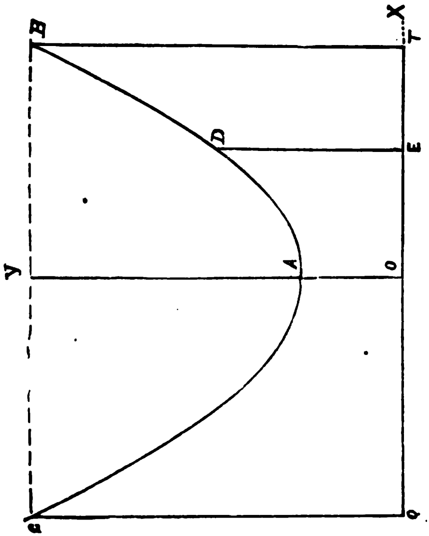



Fig. 10.

The weight of a strip a *unit in breadth* extending from A to O is then $= p =$ the weight of a unit's length of the cord. Then the part of the sheet A O D E whose weight $= w\,m\,s = p\,s$, represents the weight P on the arc A D. So A O B T represents the weight on A B, and C Q T B the whole weight on C A B. In the horizontal pull at A we have

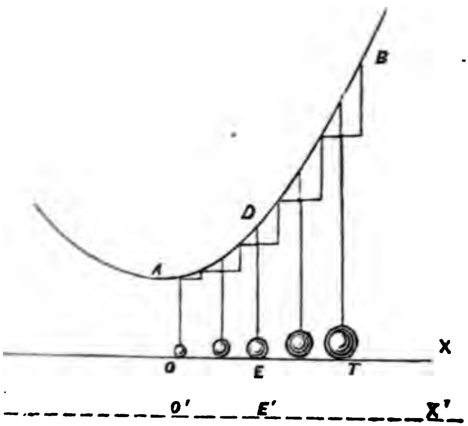
$$H = p\,m = w\,m^2 \qquad (11.)$$

and at any point D

$$T = \sqrt{H^2 + P^2} = p\sqrt{s^2 + m^2} = p\,y = w\,m\,y. \qquad (12)$$

The property above explained may be illustrated in another way.

Fig. 11.




Construct on A B (Fig. 11) a series of little triangles with all their *bases* equal. Let the weights of the little arcs constituting the hypothenuses of these triangles be represented by balls suspended by threads from the middle of each little arc. Take the length of the thread corresponding to the ball at A as $= m$; make the lengths of all the threads proportional to the weights of the balls hung to them; then the lower ends of these lines will all be on the directrix O X. That is, the *intensity* of the load on a catenary along the horizontal line (= weight on a unit of horizontal distance) varies as the *ordinates* of the catenary, when those ordinates are measured from the directrix.

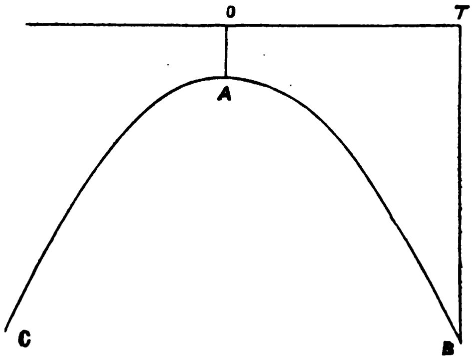
It makes no difference in the form of the curve A B (Fig. 11), to increase or diminish the weights provided the *proportion* among them is preserved. Thus we may assume the cord and the sheet C Q T B (Fig 10), to be of a different material in which a unit's length of the cord shall in weight $= p'$, and the weight of the sheet per unit of surface shall $= w'$, and A B will be unchanged. Note, however, that we cannot change the depth A O of the sheet (Fig. 10), nor the length of the lines (Fig. 11), without changing the curve, for if the lines ended in O′ X′ for instance, instead of O X then $\frac{A\ O}{D\ E}$ would *not* be equal to $\frac{A\ O'}{D\ E'}$.

*Hence*, the *modulus* ($m =$ A O) fixes the catenary, or if we assume the catenary, this determines the modulus. Thus if we assume three points, B, A, C (Fig. 10), on the catenary the distance A O is thereby determined; and if we assume A O and the point A we cannot generally assume B and C.

This often interferes with the use of the "common catonary" in the building of arches [in which case the curve is inverted, the metal sheet A O T D is replaced by a wall of uniform material, and the tension on its cord, C B (Fig. 10), is replaced by a thrust along C A B (Fig. 12)]. For we are often compelled to make the curve pass through three points, while yet the value of A O is fixed.

But this difficulty may be obviated by the use of the *transformed catenary*, which we will now discuss.

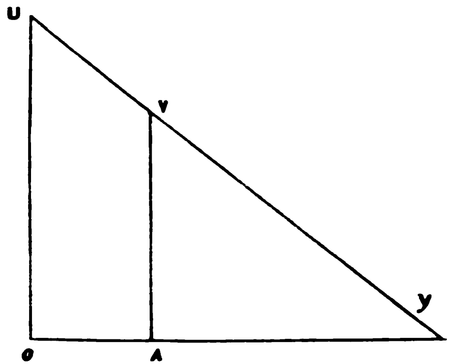
FIG. 12.




*Case III.* By the principle of *Parallel Projections*, if any cord or arched rib is balanced under a system of forces which are represented in the figure by lines, and a *parallel projection* be made of the curve of the cord or rib and of the lines representing the forces, then the new curve will represent a cord or rib that will be balanced under the forces represented by the new lines.

Imagine a cylindrical surface constructed upon C Q T B A C (Fig. 10) as a base. To simplify matters, suppose the elements of the cylinder to be perpendicular to the plane of the base. Cut this cylinder by a plane inclined to the base, and we shall get a "Transformed Catenary," and the shape of the sheet of metal under which it will be balanced; for the new curve and surface cut out by the inclined plane are the *parallel projections* of the curve C A B and the surface C Q T B A C (Fig. 10). Let this inclined plane be so placed that it shall intersect the plane of the base in the straight line C B (Fig. 10) or in one parallel to it. Then all horizontal lines (or those parallel to C B or Q T) will be unchanged in length in the parallel projection, while all vertical lines (those parallel to A O, etc.) will be lengthened in a constant ratio whose magnitude will depend upon the inclination of the cutting plane. Make a vertical section of the cylinder on the line O Y. Then if the cutting plane passes though C B we get the triangle O U Y (Fig. 13*a*) cut out of the wedge to which the cylinder reduces in this case. In the triangle, U·V is the ordinate of the vertex of the *transformed catenary* corresponding to O A in

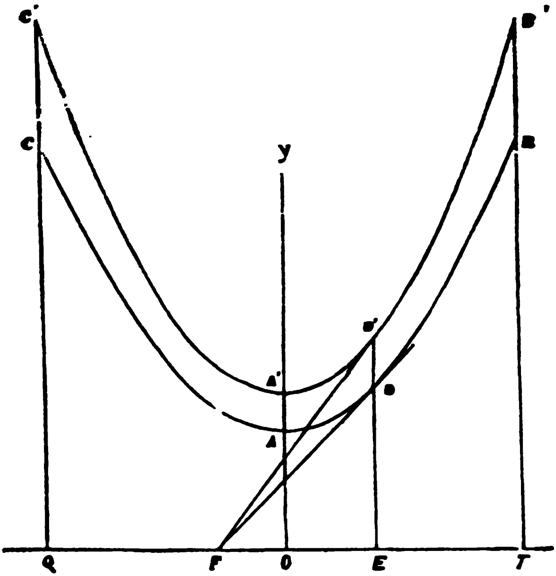
FIG 13 (*a*).




the *common catenary*, and all lines parallel to U V are evidently increased over the corresponding ones of which they are the parallel projections, in the same ratio that U V exceeds O A. Laid down in the same plane the two curves are C A B and C′ A′ B′ (Fig. 13 *b*).

It is easy to pass from a given catenary to a transformed catenary whose ordinates shall be *shorter* instead of *longer* than those of the given curve, by erecting an oblique cylinder on the given catenary and surface C Q T B, and cutting it by a plane

FIG. 13 (b .




less oblique than the base. So too, the horizontal dimensions can be changed instead of the vertical, by making the cutting plane meet the base in a line parallel to O Y, instead of in one parallel to Q T.

The equations of the curve C′ A′ B′ (Fig. 13 *b*,) are thus obtained. The *abscissas* are the same as those in C A B, but the ordinates are changed, so that (if $y'$ = general ordinate of C′ A′ B′ and $y_0$ = A′ O, the ordinate at the vertex A′)

$$y' : y :: A'O : AO :: y_0 : m.$$

$$\therefore y' = \frac{y_0}{m} . y \text{ or } y = y' \frac{m}{y_0}$$

In the equations of the common catenary substitute $y'$ for $y$ and we have the equations of C′ A′ B′.

From equation (6)

$$\frac{m}{y_0} . y' = \frac{m}{2} \left\{ E^{\frac{x}{m}} + E^{-\frac{x}{m}} \right\}$$

$$\therefore y' = \frac{y_0}{2} \left\{ E^{\frac{x}{m}} + E^{-\frac{x}{m}} \right\} \qquad (13.)$$

So equation (7) becomes

$$x = m \text{ hy. log.} \left\{ \frac{1}{\frac{y'}{y_0} + \sqrt{\frac{y'^2}{y_0^2} - 1}} \right\} \qquad (14.)$$

So equation (8) or area A′ O E D′.

$$= \int y' \, d\,x = \frac{m\,y_0}{2} \left\{ E^{\frac{x}{m}} - E^{-\frac{x}{m}} \right\} \qquad (15.)$$

etc., etc., etc.

[illegible] ... [illegible] experimental summary.

[illegible] ... [illegible] similar to those given [illegible] I.

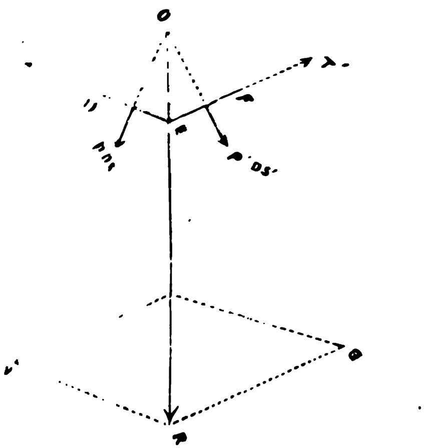
*Case IV.* So far we have discussed the forms of cords under loads *parallel* and altogether *vertical.* Let us take up the cases of loads varying in direction.

Suppose (as Case IV.) that the load be *uniform* and *normal* at *every point* to the cord. Such a load is represented in (Fig. 14), the load on each element $d\ s$ of the curve being constant and perpendicular to it.

Fig. 14.


It is first to be noted that the pull or tension on a cord under *any* load which is *everywhere normal* to it, must be *constant.* That is, the pull along the cord at A and B, and at all other points, is one and the same. That the tension at B in the cases previously discussed is greater than at A, is due to the fact that the elements of the load between A and B have in those cases *tangential* components, which go to change the value of the pull along the cord. But in the present case, the load being *everywhere* normal, there are no such tangential components, and therefore the "pull" does not change.

FIG. 15.



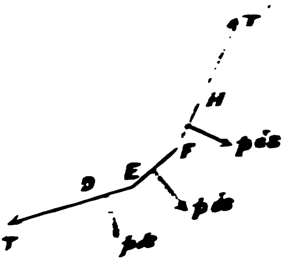
s any two adjoining elements of the

cord $d\,s$ (= D E, Fig. 15) and $d\,s'$ (= E F, Fig. 15), each of such length as to correspond to equal elements of the load. The little loads on these lines we will represent by $p\ d\ s$, and $p'\ d\ s'$. Note, that unless the load be uniform all around the cord, $d\,s$ will not be equal to $d\,s'$. The equal loads $p\,d\,s$ and $p'\ d\,s'$ being normal respectively to D E and E F, their resultant which lies in the direction O R (Fig. 15) bisects the angle between $p\ d\ s$ and $p'\ d\ s'$, and also the angle D E F between $d\ s$ and $d\ s'$, which last is the angle between the direction of the pulls T and T′ on the cord at D F. Hence the parallelogram of forces (as shown at R) will be a rhombus, or

$$N\,R = T = R\,G = T'$$

Again, take *three* elements, D E, E F, F H (Fig. 16), of the cord, each bearing the normal load $p\ d\ s = p\ d\ s = p\ d\ s$. In place of the little arcs, we use for clearness the chords of those arcs. Since the load around the whole curve C A B (Fig. 14) is supposed to be *uniform*, the arcs bearing the equal elements ($p\,d\,s$) of that load must [illegible] = [illegible] = [illegible]. [illegible] [illegible] T = T. Hence the [illegible] D E, E F, and F H will [illegible] [illegible] as in Fig. [illegible].

Fig. [illegible]

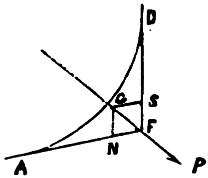



Now, every other piece of the cord containing three elements will assume exactly the same shape as D H, since each such piece must equal D H in length and must be acted on by an equal and precisely similar system of forces. Consequently, the little chords D E, etc., must constitute a *regular polygon*, and the curve in which they are inscribed must be *constant* in curvature, in other words—a *circle*.

Therefore the curve of the cord C A B (Fig. 14) is the arc of a circle.

To form the triangle of forces for any point of a loaded circle as for A D (Fig. 17),

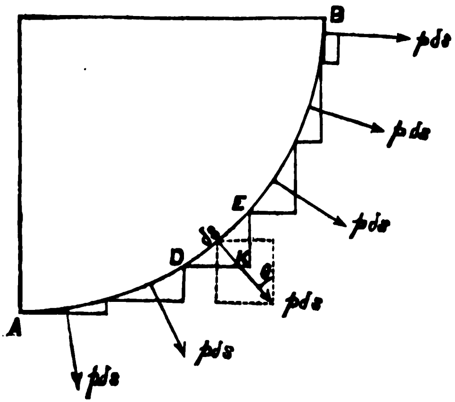
FIG. 17.




draw the tangents at the extremities A and D. From the intersection, F, of these, lay off F N = F S, to represent the equal pulls at A and D. Then the diagonal F G = the resultant of the load, and the triangle F N G or F S G represents the forces acting on A D.

It is often easier to deal with a *uniform normal* load by resolving it into its vertical and horizontal components. The load on an element D E = $ds$ of the quadrant A B (Fig. 18) is $= p\,d\,s$. The horizontal component of this load $= p\,d\,s$ sin. $\theta$, where $\theta$ = the angle made by the direction of $pds$ with the vertical (or what is the same, the angle made by the tangent of $ds$ with the horizontal). The vertical component $= pds$ $\cos.\ \theta$. Consider the horizontal component ($pds \sin.\ \theta$) with reference to the vertical space over which it is distributed. This

Fig. 18.




space is E K (Fig. 18) $= d\,s \sin.\ \theta$. Hence the *intensity* of the horizontal component

$$= \frac{p\,d\,s \sin \theta}{d\,s \sin \theta} = p.$$

So the vertical component ($p\,d\,s \cos \theta$) is distributed over a horizontal space = D K

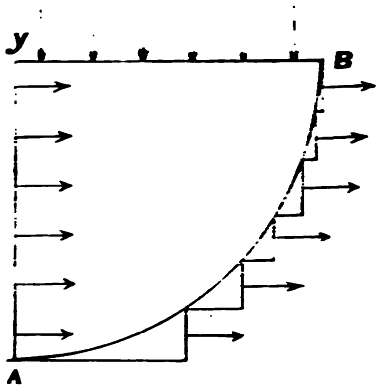
= [illegible], and therefore its intensity is

$$= \frac{[illegible]}{[illegible]} = p.$$

But $p$ = the intensity of the normal force. Hence the original normal force at each point is equivalent to a horizontal and a vertical force, at that point, of equal intensity.

If we then construct little triangles on the curve AB (Fig. 19) such that their

Fig. 19.




*vertical* sides shall be constant in length, the horizontal forces on these sides will be represented by lines of constant length. Transfer these forces in their lines of direction to A Y. A Y is the sum of all the vertical sides of the little triangles, and as the horizontal *intensity* is constant and equal to $p$, we have (if $r$ = radius of the circle) $p$ (A Y) = $pr$ = total horizontal force on quadrant A B.

Similarly, if we draw a set of triangles on A B with all their *horizontal* sides of the same length, we may see that the total vertical force on A B is

$$= p \,.\, (\mathrm{Y\,B}) = p\, r.$$

Hence,

1. The resultant of the entire normal force on the quadrant A B is equal to the resultant of a horizontal and a vertical force each of which is $= pr$.

2. Therefore in the parallelogram of forces for the quadrant (Fig. 20), F S, which represents the pull along the cord at B, is the vertical component of P, while N F = pull at A, is the horizontal component of P. Each of these forces $= pr$.

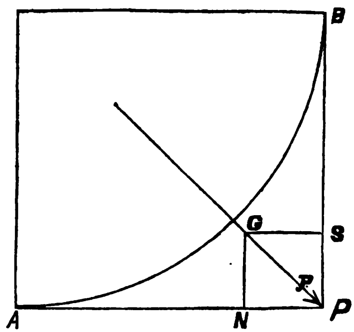
Therefore the constant pull all along the cord is $= pr$.

If we make the pull at the vertical point (B) $=$ V, we have

$$H = V = T = p\,r \quad . \quad . \qquad (20.)$$

In practice a uniform normal force exists in the case of a cylinder filled with steam, or in a vertical cylinder filled with liquid.

FIG. 20.



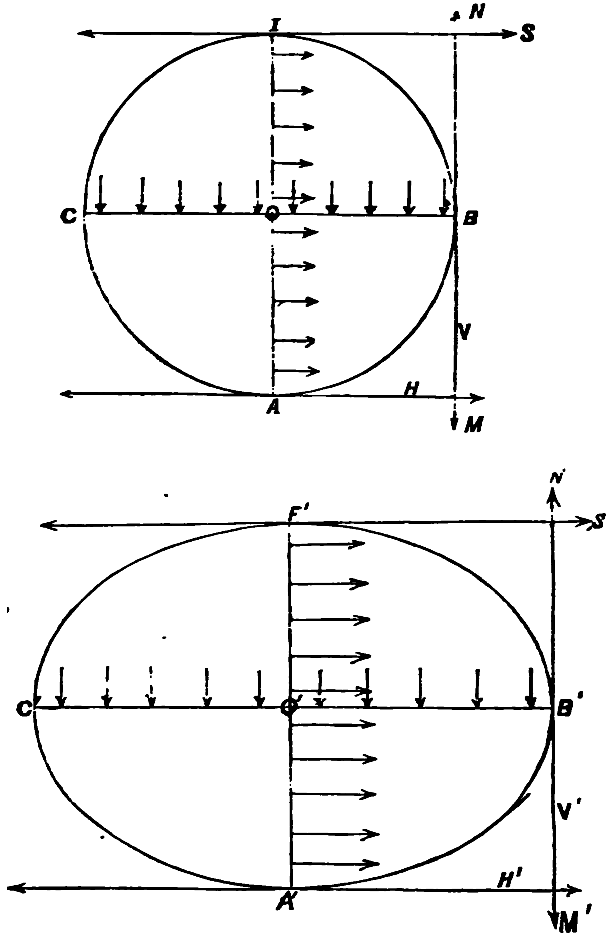

Thrust instead of tension along A B exists when the normal force pushes inwards, as in the tubes of a steam boiler or an empty vertical cylinder immersed in water. In reference to arches, this discussion has its principal value as introductory to those that follow.

*Case V.* In this case we obtain the curve and forces by parallel projections from the circle.

If we suppose a cylinder erected upon the circle (Fig. 21) as a base and cut it by an inclined plane whose line of intersection with the plane of the base shall be parallel to A I, we will get an ellipse whose vertical axis A′ I′ (Fig. 21) will = A I, and whose horizontal axis C′ B′ will be greater than C B. All lines parallel to A I will be unchanged in length, while all parallel to C B will be increased in the proportion of C′ B′ to C B. Now, by the principle of parallel projections, the ellipse, which is the parallel projection of the circle, will be balanced under the forces which are the parallel projections of those under which the circle is balanced.

As we have seen, the circle is the curve assumed by the ring under a uniform horizontal and vertical force at each point of the same kind, and equal in intensity; for such

FIG. 21.



a system of forces is equivalent to a constant normal force around the curve. For convenience, these forces are represented in Fig. (21) along the two diameters, each little line representing the force on a unit of distance. The pull around the ring is of course tangential to it, and is everywhere the same ($=pr$). This pull is represented at A and B by the arrows there.

In the ellipse, the *vertical* lines being unchanged, the total vertical force on the elliptic ring (= the sum of all the little vertical lines) is the same as it was in the circle, and if we call the vertical force on a quadrant V (= B M) for the circle and V′ (= B′ M′) for the ellipse, we will have

$$V = V' \quad . \quad . \quad . \quad (21.)$$

Notice, however, that in the ellipse the force V′ is distributed over the distance O′ B′ and not over a distance = O B. Hence the *intensity* of the force V′, or the amount of that force on each unit of distance, is not the same as in the circle. In the ellipse (Fig. 21) each little vertical line represents, therefore, the force on a distance greater than a unit. Let $O'B' = c\,OB$. Then to obtain the *intensity* of $V'$, divide it by the space over which it is distributed. Thus, let

$$p_v = \frac{V}{OB} \text{ and } p_x = \frac{H}{AO}$$

represent the vertical and horizontal intensities in the circle. We have already seen that in the circle

$$p_v = p_x = p.$$

Let $p'_v$ and $p'_x$ represent the vertical and horizontal intensities in the ellipse. Then

$$p'_v = \frac{V'}{O'B'} = \frac{V}{c\,.\,OB} = \frac{p_v}{c} \qquad (22.)$$

The lines representing the "pulls" at B and C (as B N) are also unchanged. Hence the pulls at those points in the elliptic ring are the same as in the circular; that is they are equal to $V' = V$.

The *horizontal* lines are all increased in length in the ratio $1 : c$. Hence the sum of the lines representing the horizontal force on a quadrant of the ellipse (as $I'S'$) is greater than the corresponding line (I S) in the circle in the above ratio. Therefore if

H' = the horizontal force on the elliptical quadrant,

$$H' = c . H \qquad . \qquad . \qquad (23.)$$

The length over which this force H is distributed (A' O'), does not change, however, and hence the little horizontal lines in both figures represent the force on a unit of distance. Hence the *intensity* of the horizontal force in the ellipse has increased just as the length of the lines, or from the equation

$$p'_x = \frac{H'}{A' O'} = \frac{c . H}{A O} = c\, p_x \qquad (24.)$$

The horizontal pull in the ring at A' or I' being equal to the horizontal force on a quadrant is

$$H' = c . H = c . V = c . V' \qquad . \qquad . \qquad (25.)$$

Hence the "pull" around the ellipse is not constant as it was in the circle. The pulls at B' and A' are as

$$V' : H' : : 1 : c.$$

But

$$A' I' : C' B' : : 1 : c.$$

Therefore,

1. The pulls in an elliptical ring are as the axes to which they are parallel.

Again the intensities in the ellipse are

$$p'_y : p'_x : : \frac{p_y}{c} : c\,p_x : : \frac{1}{c} : c : : 1 : c^2$$

And

$$(A'\,I')^2 : (C'\,B')^2 : : 1 : c^2$$

Therefore,

2. The *intensities* of the forces in an ellipse are as the *squares* of the axes to which they are parallel.

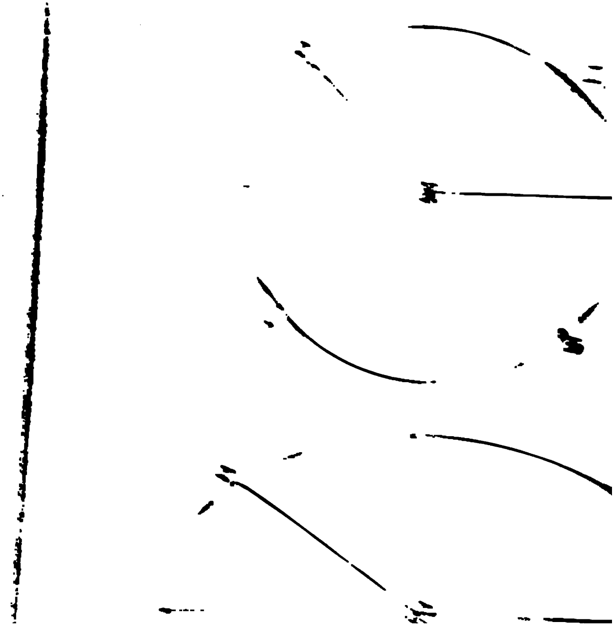
From this proportion we have

$$c = \sqrt{\frac{p'_x}{p'_y}} \quad . \quad . \quad . \quad (26.)$$

It will be noted in the elliptic ring that the resultant of the little horizontal and vertical loads at any point is not *normal* to the curve except at the extremities of the axes.

Let us determine the pulls and the relations between the forces at other points besides the extremities of the vertical and horizontal axes of the ellipse.

In the circle (Fig. 22) if we resolve the forces along any two rectangular axes as $A_1\,I_1$ and $C_1\,B_1$, we shall have evidently the same relations between them as when resolved along a vertical and horizontal axis.



Now the three parallel lines, viz., the diameter, $A_1 I_1$, and the tangents at $C_1$ and $B_1$, are projected in the ellipse into three *parallel* lines, viz.: $A'_1$, $I'_1$, and the tangents at $C'_1$ and $B'_1$. Similarly $C_1$, $B_1$, and the tangents at $A_1$ and $I_1$ continue parallel in the ellipse. Hence *rectangular* diameters of the circle become *conjugate* in the ellipse. The lines representing the forces perpendicular to $C_1 B_1$ in the circle become parallel to $O' I'_1$ in the ellipse, and are changed in length just as $O' I'_1$ is changed from $O I_1$. So the forces which are parallel to $C_1 O$ in the circle become parallel to $C'_1 O'$ in the ellipse, and vary as $C'_1 O'$ does from $C_1 O$.

Let $O' I'_1 = r'$ and $O C'_1 = r''$ and let the total force parallel to $O' I'_1$ on a quadrant (such as $C'_1 I'_1$ or $I'_1 B'_1$) of the ellipse be $= V_1$ and that parallel to $O' B_1'$ be $= H_1$. Then if $r =$ radius of the circle, we have (since the force on a quadrant of the circle as $C_1 I_1$ is $= H = V = T$)

$$\left.\begin{array}{ll} H : H_1 :: r : r'' & \therefore H_1 = \dfrac{H . r''}{r} \\ V : V_1 :: r : r' & \therefore V_1 = \dfrac{V\, r'}{r} = \dfrac{H . r'}{r} \\ \therefore H_1 : V_1 :: r'' : r' & \end{array}\right\} \quad (27.)$$

$H_1$ is equal to the pull along the ring at $A'_1$ or $I'_1$, and $V_1$ is that at $C'_1$ and $B'_1$.

Hence *proposition* 1 may be applied generally to all conjugate diameters in the ellipse; that is,

3. The total pulls along the ring at the extremities of any two conjugate diameters, are as the diameters to which they are parallel.

Again, the *intensities* being equal to the total loads divided by the surfaces over which they are distributed, let

$p'_{y_1}$ — intensity of load parallel to $O'\,I'_1$,
$p'_{x_1}$ " " " $C'_1\,O'$

Then

$$\left.\begin{aligned} p'_{y_1} &= \frac{V_1}{O'C'_1} = \frac{V\,r'}{r \cdot r''} = p_y\,\frac{r'}{r''}\,. \\ p'_{x_1} &= \frac{H_1}{O'\,I_1'} = \frac{H\,r''}{r\,.\,r''} = p_x\,\frac{r''}{r'} \\ \therefore p'_{y_1} : p'_{x_1} &:: p_y\,\frac{r'}{r''} : p_x\,\frac{r''}{r'} :: \frac{r'}{r''} : \\ &\quad \frac{r''}{r'} :: r'^2 : r''^2 \end{aligned}\right\} (28.)$$

Hence for *proposition* 2, we may read,

4. The *intensities* of a pair of conjugate loads are to each other as the *squares* of the

conjugate diameters to which they are respectively parallel.

To pass from one set of conjugate forces on the ellipse to another; let

$p'_{x_1}$ and $p'_{y_1}$ be the intensities parallel to one set of conjugate diameters.

$H_1$ and $V_1$ be total pulls parallel to same set of conjugate diameters.

$r'' \; r'$ be the conjugate semidiameters.

Also let

$$p'_{1x_1}, \; p'_{1y_1}, \; H'_1, \; V'_1, \; r''_1, \; r'_1$$

be the corresponding quantities for the other set. Then

$$\left.\begin{array}{ll}
p'_{x_1} = p_x \dfrac{r''}{r'} & \therefore \; p_x = p'_{x_1} \dfrac{r'}{r''} \\
p'_{1x_1} = p_x \dfrac{r''_1}{r'_1} & \therefore \; p'_{1x_1} = p'_{x_1} \dfrac{r' \, r''_1}{r'_1 \, r''} \\
\text{Also,} & \\
H_1 = \dfrac{H . r''}{r} & \text{or } H = \dfrac{H_1 \, r}{r''} \\
H_1' = \dfrac{H \, r''_1}{r} & \therefore \; H'_1 = \dfrac{H_1 \, r''_1}{r''} \\
\text{Similarly} & \\
p_1'_{y_1} = p'_{y_1} \dfrac{r'' \, r'_1}{r' \, r''_1} & \\
V_1' = V_1 \dfrac{r'_1}{r'} &
\end{array}\right\} \quad (29)$$

The ellipse (Figs. 21 and 22) is the form assumed by a cord under a load composed of horizontal and vertical components which are constant along the horizontal and vertical lines, but which differ from each other in intensity.

The diameter C′ B′ of the ellipse (Fig. 21) might have been made *shorter* instead of *longer* than that of the circle, if required.

*Cor.*—If one set of the forces are vertical and the other not horizontal, but inclined at an angle to the horizon (Fig. 23), we still have an ellipse, the directions of the forces giving the directions of two conjugate diameters ($A'_1\,O'$ and $B'_1\,O'$). Then, if $p'_{\underset{1}{x}}$ = the intensity of the inclined force and $p'^{\,y}_1$ = intensity of the vertical force, we have by proposition 4,

$$p'_{x_1} : p'_{y_1} :: (B_1'\,O')^2 : (A_1'\,O')^2$$

So from proposition 3, if $V_1$ = pull along the cord at $B_1'$ or $C_1'$ and $H_1$ = that at $A_1'$

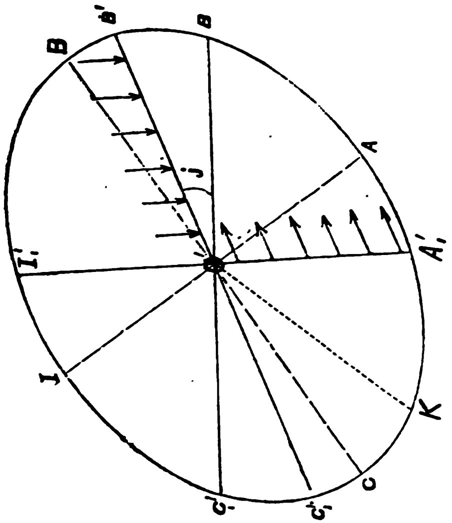
$$H_1 : V_1 :: B_1'\,O' : A_1'\,O'.$$

From the first of these propositions we have the ratio of the conjugate diameters;

and from the second we find the pulls at the extremities of those diameters.

Knowing two conjugate diameters and the angle ($90°-j$) between them we can readily obtain the ellipse.

FIG. 23.




To obtain the pulls at the extremities of any diameter, such as $C_1\,B_1$.

This is merely passing from one set of conjugate diameters to another and equation (29) gives the pull at $B_3$, for instance, as

$$T_1' = T_1 \frac{O' K}{O' A_1'}$$

($O' K$ being conjugate to $C_1 O' B_1$), etc., etc.

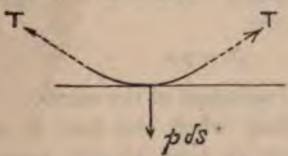
An important fact is now to be noted. Whenever the load on a cord is entirely *normal* to it, at that point the pull along the cord is equal to the intensity of the normal load multiplied by the radius of curvature.

For the cord *at that point* is similarly situated to a circular cord of the same curvature and under a load of the same intensity.

Thus, in the ellipse (Fig. 21) the action of the load at the extremities of the axes is entirely *normal*, for at A′ and I′ the horizontal component of the load vanishes and leaves only the vertical, which, *at these points*, is *normal* to the curve. So at C′ and B′ only the horizontal load has value, and its action is there normal to the curve.

Consider the elementary arc, $ds$, at A′, for instance, which is subjected to this normal load. It is balanced under the equal pulls $T = T'$ (Fig. 24) coming from the adjoining parts of the cord, and the normal load *pds*, which gives it its curvature. Imagine a circle under a constant normal force of intensity $= p$. Take an equal little arc $d\,s$ of it, loaded with a normal load $=$ *pds*. Then, if it be acted on at its two ends by tensions $= T = T'$, it is evident that it will have the same curvature as the arc of the ellipse; or, conversely, if it has *the same curvature*, the pull around the circle *must* be $= T = T'$.

FIG. 24.




Hence, having given the load on the curve at any point where it is *normal*, we determine easily the pull along the cord at that point. For, in the circle,

$$H = V = T = p_x\, r = p_y\, r = p\, r$$

and in the ellipse at A′

$$H' = p'_v \rho$$

Where $\rho$ = radius of curvature. If A′ O′ = $r$ and O′ B′ = $c\,r$ in the ellipse (Fig. 21) we have at A′

$$\rho = \frac{c^2\, r^2}{r} = c^2\, r.$$

$$\therefore H' = p'_v \,.\, c^2\, r = \frac{p_v}{c} c^2\, r = c\, p\, r = c\, H. \quad (30.)$$

So in the parabola under uniform vertical loads (Case I.) we have seen that H = 2 $p$ $m$ (Rankine's C. E. p. 165). But H = $p\,\rho$ = 2 $p$ $m$ (since $\rho$ = 2 $m$ at the vertex).

If the load be *everywhere* normal to the cord, the above equation will apply to every point, or

$$T = p\,\rho$$

be a general equation of the curve.

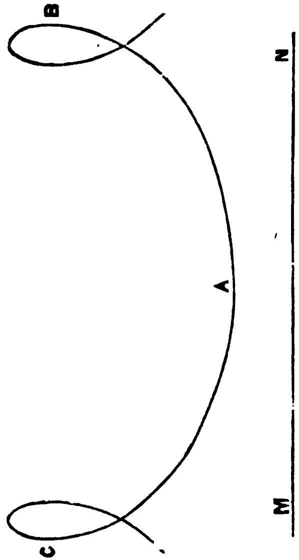
And further, when the load is everywhere normal we have already seen that the pull along the cord must be *constant*, as there is no tangential force to change it. Hence.

$$T = p\,\rho = \text{a constant.} \qquad (31.)$$

When the load $p$ is constant, of course, $\rho$

must be constant too, and we have the circle already discussed. When $p$ varies, $\rho$ must *vary inversely* as $p$.

Fig. 25.




*Case VI.*—If $p$ increases in value just in proportion to the distance of the points of the cord above a horizontal line M N (Fig.

25), the cord assumes the shape of the hydrostatic arch. This curve possesses geometrically the loops shown in the figure and may be extended indefinitely, but for our purpose it is evidently only necessary to discuss that part between the points C and B (Fig. 26) where the tangents are vertical.

Taking L (Fig. 26) for the origin, if the intensity of the load then be $y_0$ (= A L) multiplied by a constant, or $wy_0$, then at any other point it is $= wy$.

Hence the equation of the curve is

$$T = p\,\rho = w\,y\,\rho = w\,y_0\,\rho_0 = \text{a constant}$$

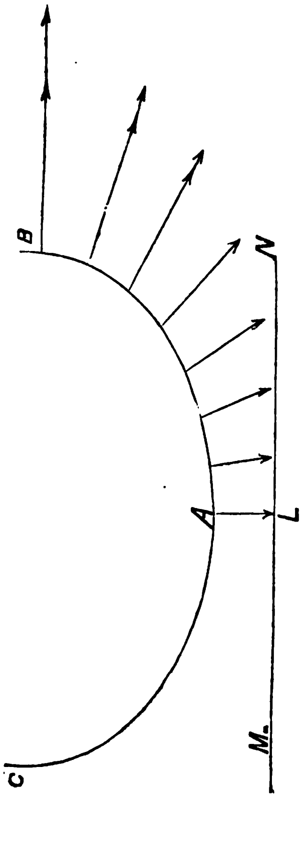
($y_0$ and $\rho_0$ are the values of the ordinate and radius of curvature at A).

Let us resolve the normal load on C A B as we did in the circle, into its horizontal and vertical components. As was the case in the circle, these will be *for each point* equal in *intensity* to each other and also to the *normal* force, or

$$p = p_x = p_y.$$

But these quantities are no longer constant (as in the circle) all along the curve, but vary from point to point.

**Fig. 26.**



If we form the parallelogram of
any arc A D (as in Fig. 27) the si
F S, since H = T = a constant,
must represent the resultant of
load on A D both in amount and
The vertical component, F E
equal to the vertical componen
S F, or

Vertical load on A D = T sin $i$

At B the vertical load = T

(since $i = 90°$ there).

So the horizontal component of
load on A D is G E, and since

N F = G S = G E + F X

we have horizontal load on

A D = G E = N F − F X = H − H c
H (1 − cos $i$),

At B, $i = 90° \therefore$

Horizontal load on A B = H

On the arc D B

Horizontal load = H − H (1 − cos $i$) =

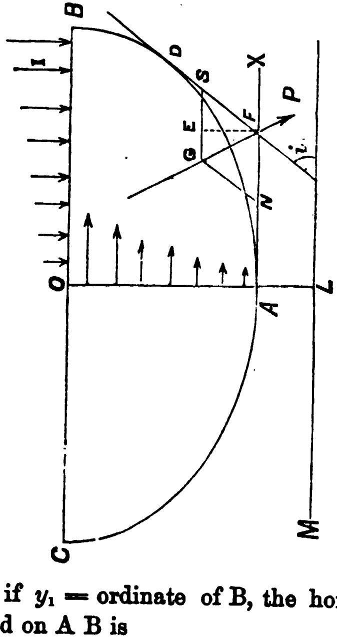
The *vertical* load on A D may
expressed

$$H \sin i = \int_0^x p_y \, dx = w \int_0^x y \, dx = w \, y_0 \, p$$

The *horizontal* load thus

$$H\,(1-\cos i) = w\, y_0\, p_0\, (1-\cos i) = \int_{y_0}^{y} p_x\, d\, y = w\int_{y_0}^{y} y dy = w.\frac{y^2-y_0{}^2}{2} \qquad (33.)$$

FIG. 27.




And if $y_1$ = ordinate of B, the horizontal load on A B is

$$H = w\frac{y_1^2 - y_0^2}{2} \qquad (34.)$$

For formula for radius of curvature see Rankine, C. E., p.

The equation $T = H = w\, y_0\, \rho_0 = w\, y\, \rho$, enables us to solve problems similar to those under the parabola.

*Case VII.*—If we construct a curve from the last one by using the same ordinates and by changing all the abscissas in the ratio $c : 1$, so that the new co-ordinates of a point shall be $y$ and $c\,x$, and at the same time change the *horizontal* forces in the same proportion, leaving the *vertical* ones unchanged; the new curve and new system of forces so obtained will evidently be parallel projections of the former, and will be balanced. This new curve C′AB′ (Fig. 28) is the "Geostatic," and bears a relation to the "Hydrostatic" strictly analogous to that between the ellipse and circle.

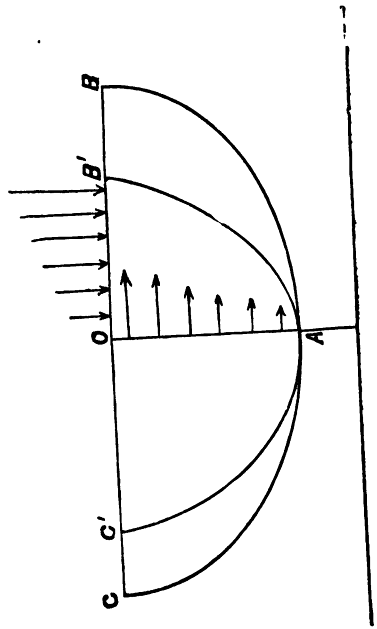
Hence,

The total vertical load on A B′ = V′ =
V = pull along cord at B′.
tal horizontal load on A B′ = H′ =
$c$ H = pull along cord at A′. (35.)

The *intensities* are

$$\left.\begin{aligned} &\text{For } \textit{vertical} \text{ load } p'_y = \frac{V'}{O\,B'} = \frac{V}{c.OB} = \frac{p_y}{c} \\ &\text{For horizontal load } p'_x = \frac{H'}{O\,A} = \frac{c\,H}{O\,A} = c\,p_x \end{aligned}\right\} \quad (36.)$$

Fig. 28.




(V H $p_x$ and $p_y$ referring to the hydrostatic curve.)

zontal load should be also uniform, and of intensity equal to that of the vertical load.

But generally: Let C A B (Fig. 29) be some assumed curve, and let the vertical load be known in amount and distribution. Making some changes in the signification of the letters heretofore used, now let

$V$ = vertical load on any arc A D.
$V_1$ = vertical load on the semi-cord A B.
$H$ = horizontal load on any arc A D.
$H_1$ = " " half-cord A B.
$H_0$ = pull along cord at A (the quantity heretofore denoted by H).
$p_x$ and $p_y$ = the horizontal and vertical intensities as heretofore.
$p_0$ = value of $p_y$ at the point A.
$\rho_0$ and $\rho_1$ = radii of curvature at A and B.

The vertical load on an arc A D is

$$V = \int_0^x p_y \, dx \quad . \quad . \quad (39.)$$

Again at the horizontal point A, the vertical projection of the element of the curve being = zero, the load is entirely vertical, and consequently at that point is *normal* to the curve. Hence the pull along the cord at A is

$$H_0 = p_0 \rho_0.$$

To discuss the forces upon an arc A D. Draw tangents at A and D. They meet at F (Fig. 29), through which point the resultant of the total load on A D must pass. The *vertical load* is also = the vertical component of the pull along the cord at D, for these two forces, being the only vertical ones connected with A D, must needs balance each other. Therefore,

Lay off F N = $H_0$. Lay off F E vertical and $=\int_0^x p_y\, dx$. Complete the rectangle F E S X. The pull along the cord at

$$D = F S = F E \operatorname{cosec} i = V \operatorname{cosec} i \quad . \quad . \quad (40)$$

Also,

$$S E = F X = V \cot i = \text{horizontal compound of pull along the cord at D} \quad . \quad . \quad (41)$$

But the horizontal pull at A is

$$H_0 = F N = G S.$$

$$\therefore G E = H_0 - V \cot i = H = \text{resultant of horizontal load on A D} \quad . \quad . \quad (42.)$$

The *intensity* of this horizontal load may be expressed thus

$$p_x = \frac{d H}{d y} = -\frac{\delta (V \cot i)}{d y} = -\frac{\delta \left( V \frac{d x}{d y} \right)}{d y} \quad (43.)$$

At B the vertical load $= V_1$. Let this be represented by B K (Fig. 29). If the cord be itself vertical at that point, B K $=$ $V_1$ will be equal to the pull along it at B. If the cord is inclined as in the figure, draw its tangent at B, and

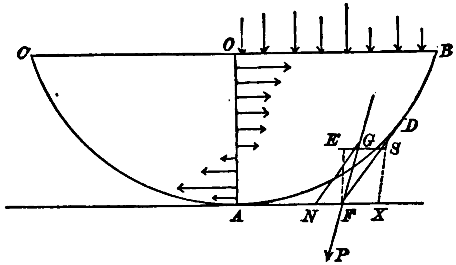
B L $=$ B K cosec $i_1 = V_1$ cosec $i_1 =$ pull along the cord.

and

K L $=$ B K cot $i_1 = V_1$ cot $i_1$ $=$ horizontal component of this pull.

$H_0 - V_1 \cot i_1 = H_1$ $=$ resultant of entire horizontal load on A B.

FIG. 30.




It may often happen that S E $=$ V cot $i$ $=$ horizontal component of the pull along the cord at D (Fig. 30) is *greater* than G S

$= F\,N = H_0 =$ horizontal pull along the cord at A. In such cases $G\,E = H_0 - V \cot i$ is negative, which indicates that the horizontal load between A and D, for at least a part of this distance, must be contrary in direction to that heretofore discussed; that it must exert an *inward pull* instead of an *outward one* (Fig. 30). If this "inward pull" were removed or replaced by an outward one, the curve would evidently be *flattened* about A.

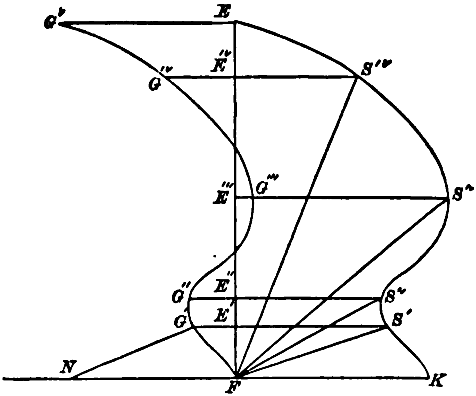
We may illustrate geometrically, the relation between the forces in all parts of A B.

The vertical load and curve being given draw $F\,E^{v}$ (Fig. 32) = the total vertical load on A B, and lay off on it

$F\,E' =$ vertical load on the arc A D′.
$F\,E'' =$ " " " A D″, etc.

Draw a horizontal line at F and lay off F N and F K, each $= H_0$ = pull at A. Draw through F lines parallel to the tangents at D′ D″ D‴, etc., and through E′ E″ E‴, etc., lines parallel to the horizon. Then the oblique lines F S′, F S″, etc., represent the pulls along the cord at D′ D″, etc., while E′ S′, E″ S″, etc., represent the horizontal components of these pulls. Lay off from each point S′ S″, etc., horizontal lines, each equal to F N, and draw through

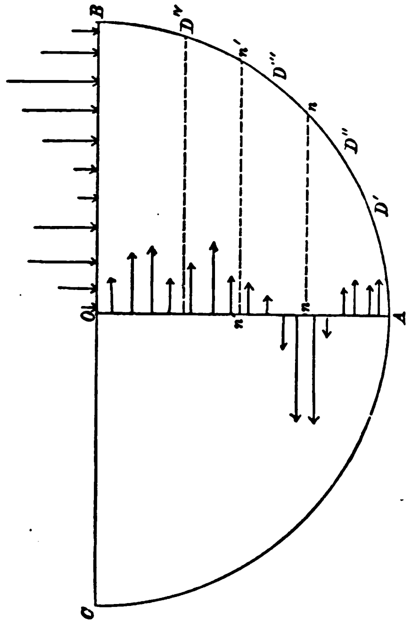
Fig. 32.




the points G′ G″, etc., thus obtained, a curve. It will evidently be similar to that drawn through K S′ S″, etc., and the line G′ E′ will represent the resultant of the horizontal load that must be distributed along the curve from A to D′; G″ E″, the resultant of the horizontal load between A and D″, and so on.

(Fig. 32) is really formed from the parallelogram of forces for the arcs A D′, etc.;

Fig. 31.




this parallelogram being at D′= F N G′ S′, in which E′ S′ is the horizontal component of the pull at D and $E' G'$ = the resultant of the horizontal load on A $D'$.

As the abscissas of the curve F $G'$ $G''$, etc., increase to the left of F $E^{v}$ from the point F to $G''$ (which correspond to $D''$ on the curve), the horizontal load acts outward on the arc A $D''$. The abscissas then diminish to $G'''$. Hence between $D''$ and $D'''$ on the curve, the horizontal load must act inwards as shown in (Fig. 31). From $G'''$ the abscissas increase until we reach $G^{v}$. Hence the horizontal load acts outward throughout the remainder of the cord. The points $n$ and $n'$ correspond to those arcs on which the resultant of the horizontal load is zero. Thus on the arc A $n$ the *negative* horizontal load is just equal to the *positive*, and hence their sum = zero. So on the arc A $n'$.

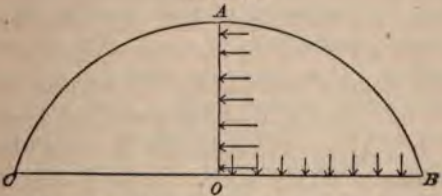
Note that the abscissas of the curve F $G'$ . . . $G^{v}$ are not the *intensities* of the horizontal loading, but that each such abscissa represents the *algebraic sum* of the entire horizontal load between A and the point to which the abscissa corresponds. The *intensity* in question has already been ˈıown to be

$$p_x = \frac{d\,\mathrm{H}}{dy}.$$

In this expression $d\,\mathrm{H}$ = the difference of two neighboring abscissas of the curve $\mathrm{F\,G'} \ldots \mathrm{G^v}$; as for instance, $d\,\mathrm{H} = \mathrm{G'\,E'} - \mathrm{G''\,E''}$. And $dy$ = vertical projection of the arc $\mathrm{D'\,D''}$ of the cord to which the above corresponds.

## ARCHES.

FIG. 33.



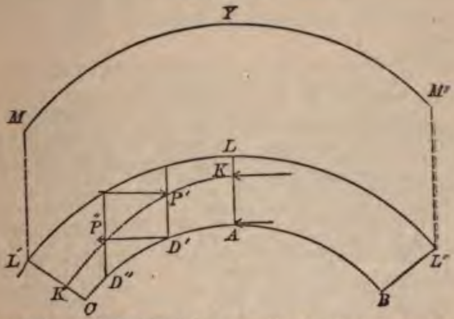

Let us imagine the curve of the cord to be reversed, and the cord itself to be replaced by a thin metal strip, which like the cord shall be practically without transverse stiffness, but, *unlike* the cord, shall be able to resist a compressive force in the direction of its length at every point. Let the loads be distributed as heretofore, except that where there are horizontal components of the load, these should act *inward*, where upon the cord they acted *outward*, and *vice versa*. We then have what is called a "linear arch or rib"; and the curve assumed by it will be identical with that of the cord under equal and similarly distributed loads. If the loading is changed in distribution, the rib will change in shape just as the cord would do under similar circumstances.

In practice there are no "linear arches," but the discussion of them enables us to determine the form of equilibrium for real arches. If we know the form that a *linear arch* would assume under a given load, we can find the "line of pressures" in the real arch. This line and the value of the thrusts at all its points enable us to solve the problems that arise in arch building.

1. Suppose, for instance, we desire to construct an arch to bear a *uniform vertical load*, such as that discussed in *Case I*. The shape of the linear rib under such a load is a parabola. We then as,

1° Step. Assume this curve for the intrados C A B (Fig. 34). If the arch and the load be of homogeneous material, the shape of the *extrados*, or outside of the load,

Fig. 34.




will be M Y M′, the vertical distance between O A and M Y being constant.

2° Step. Is to determine the depth A L of the keystone. This depth is always greater than necessary simply to prevent the crushing of the material of the arch under the thrust at the crown. Prof. Rankine's empirical rule derived from the best examples is to make the depth of the keystone in feet

$$\left.\begin{array}{l}\text{In single arches}\\ =\sqrt{.12 \times \text{radius of curva'e at the crown.}}\\ \text{In arches of a series}\\ =\sqrt{.17 \times \text{radius of curva'e at the crown.}}\end{array}\right\} \quad (46.)$$

3° Step. Determine whether the "line of pressures" can lie in the "middle third" of the ring of voussoirs. It should be restricted to the *middle third* to prevent the voussoirs tending to open at any of the joints.

We can test this as follows:

Suppose the voussoirs to be constant in depth all around the arch as in (Fig. 34.) Consider any part of the arch included between the vertical plane (A L) at the crown, and a vertical plane at any other point, as D′ P′. The calculated horizontal thrust along the linear rib, which coincides in shape with the soffit C A, is indicated by the arrow with its head at A. Let the *horizontal thrust* of the rib at D′ be indicated by the arrow with its head at D′ pointing in an opposite direction to that at A. At the crown take A K not greater than $\frac{2}{3}$ A L. Imagine a left-handed couple applied to A L in the vertical plane of the arch, whose force = H = the thrust at A, and whose lever-arm = A K. Apply an equal and opposite couple on the plane D′ P′, with a force H′, equal to the horizontal thrust of the rib at D′. Its lever-arm D′ P′ must then

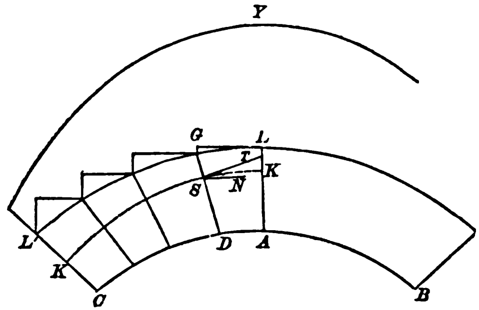
$$= \frac{H . A K}{H'}$$

In the parabola $H = H'$ ∴ $D' P' =$ A K. These couples being equal and opposite do not change the conditions of equilibrium of the section of the arch L D′, but they transfer the line in which the thrust acts from A D′ to K P. We can repeat the process as often as we choose by taking parts L D″, etc.; and if the curve drawn through the points K P′ P″, etc., lies within the *middle third* of the arch-ring, the arch is sufficiently stable.

In the case before us, the horizontal thrust being constant for every point of the rib C A, the lever-arms D′ P′, D″ P″, etc., are also equal, and therefore the "line of pressures" K K′ is merely the parabola raised vertically a distance = A K. If K K′ does not lie in the *middle third*, a slight increase in the voussoirs, especially towards the springing, will usually remove the difficulty.

4° Step. The joints between the voussoirs, such as D′ G (Fig. 35) are usually made normal to the soffit A C, but whether this be done or not, the direction of G D′ must be such that at S, where the line of pressures cuts it, the angle included between

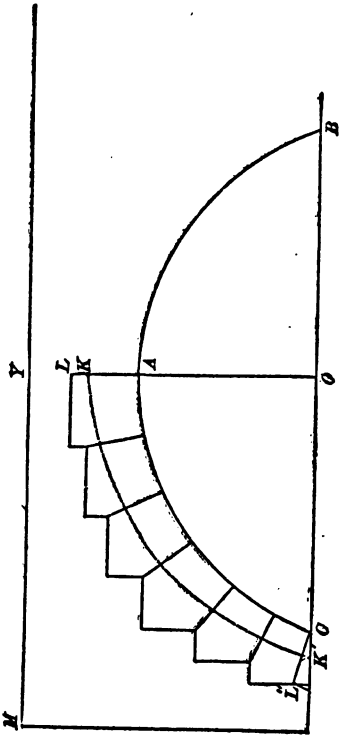
Fig. 35.




S N (the normal to G D′) and S T (the tangent at S to K K′) may be less than the angle of friction of the material of the voussoirs. The best possible direction for the joints D′ G, etc., would be to make them perpendicular to K K′.

The horizontal component of the thrust (H) along the curve of pressures in a parabolic arch, is, as we have seen, constant; but the thrust along the curve (T) increases

FIG. 36.




from A to C, and its value at any point may be determined by the formulæ in Case I.

Parabolic stone or brick arches are not common, because it is rare to have such a distribution of the load as that supposed above.

2. But if we reverse the curves discussed under Cases II. and III., we have a form of arch much more frequently applicable.

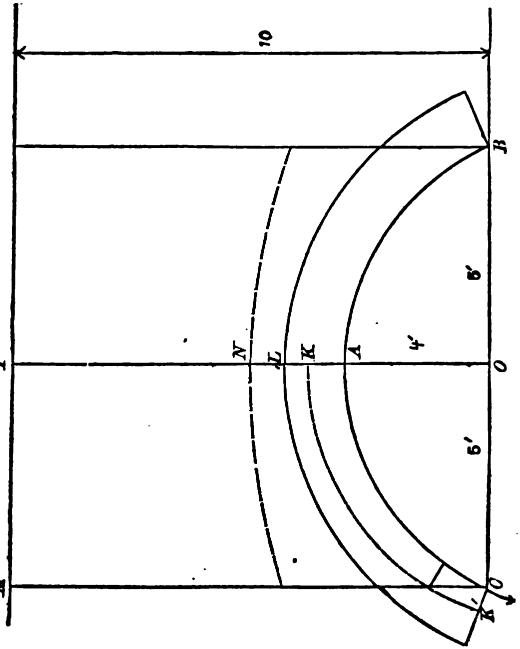
Thus, suppose the arch and its backing to be homogeneous, and that the *extrados* of this loading is horizontal (M Y), and suppose the action of the load to be entirely vertical. Then the arch and its backing are similar to the metal sheet and the cord discussed in the cases just referred to, and therefore the form of the *linear rib* under such a load will be a catenary or transformed catenary—usually the latter.

Assume this curve for the soffit C A B (Fig. 36); determine the depth A L; the line of the pressures K K′; and the direction of the joints; as in the last case. In this case as in the parabola, H is constant, and hence the curve of pressures is merely the curve C A raised vertically through a distance = A K.

*Example.* Let the data for a required

arch be (Fig. 37) span C B = 10′; rise O A = 4′; height of extrados M Y above

Fig. 37.




springing at C = 10′. Let the arch and

brickwork be of solid brickwork whose weight $w$ per cubic foot = 112 lbs.

The equation of the transformed catenary passing this C A B is

$$y = \frac{y_0}{2}\left\{ E^{\frac{x}{m}} + E^{-\frac{x}{m}} \right\}$$

Where $y_0$ = A Y = 6′ (the origin being at Y and the axis of abscissas horizontal).

First find $m$, the modulus of the corresponding common catenary. By Eq. (14.)

$$m = \frac{x'}{\left\{ \frac{y'}{y_0} + \sqrt{\frac{y'^2}{y_0^2} - 1} \right\}}$$

At the point C $x' = 5$ ft. and $y' = 10$ ft.

$$\therefore\ m = 4.54 \text{ ft.} = \text{Y N.}$$

Then determine points of the curve, thus for

$x = 1$, $y =$ for $x = 2$, $y =$ for $x = 3$, $y =$ for $x = 4$, $y =$ etc.

Describe the curve through these points.

The thrust at the crown A is (for a unit of length of the arch)

$$H = w\,m^2 \text{ from Eq. (16).}$$

$$\therefore\ H = (112)\,(4.54)^2 = 2308.3 \text{ lbs.}$$

From Eq. (15) area A Y M C =

$$\frac{m y_0}{2}\left\{E^{\frac{x}{m}} - E^{-\frac{x}{m}}\right\} = 36.32 \text{ sq. ft.}$$

Weight of load A Y M C = P = (112) (36.32) = 4067.84 lbs.

From Eq. (18) thrust at C = T = $\sqrt{P^2 + H^2}$ = 4677.1 lbs.

Inclination at C . Tan $i_1 = \frac{d\,y_1}{d\,x_1} = \frac{y_0}{2\,m}$

$$\left\{E^{\frac{x'}{m}} - E^{\frac{x'}{m}}\right\} = 1.77.$$

$$\therefore\ i_1 = 60^\circ\ 32'.$$

The formula for depth of keystone will be satisfied by making the depth of the arch A L = length of one brick = 9″, for this gives 9″ × 12″ = 108 square inches to bear the thrust H = 2308.3 lbs., or T= 4677.1 lbs. The latter is the greatest thrust in the arch.

It is easy to see that K K′ will be in the middle third, for even at C the distance of the point of the curve K K′ vertically over C, from the nearest point of C A, is approximately

$$6 \times \cos\,(90^\circ - 60^\circ\ 32') = 5'' +.$$

The *extrados* of the transformed catenary need not be the directrix M Y; it *may* be another transformed catenary *provided these catenaries have the same directrix.*

To illustrate: suppose the weight of a

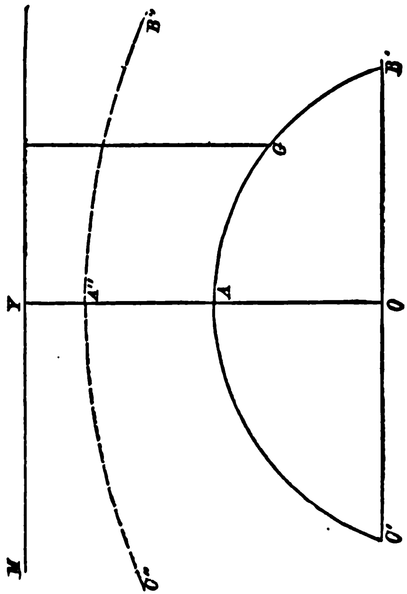



Fig. 38.

unit of the material between C A B and M Y $= w$. Then the intensity of vertical pressure at any point G of C A B (Fig. 38) is $=$ $w\,y$. If a heavier building material were used this vertical pressure could be brought upon G by a less height of it. Let this heavier material have a weight per unit $=$ $w'$ and let

$$w' = \frac{2}{3}\, w.$$

Then a column of the heavier material over G and of a height $= \frac{2}{3}\, y$ would give the same pressure as the whole column of the lighter, or

$$w\,y = \frac{2}{3}\, w'\,y \quad . \quad . \quad (47.)$$

At each point of C A B (Fig. 38) lay off two-thirds of the vertical ordinate, and through these points draw C″ A″ B″. The upper surface of the load may have this form, and yet C A B still be the shape of the linear arch balanced under the applied forces. The equation of C A B being

$$y = \frac{y_0}{2} \left\{ E^{\frac{x}{m}} + E^{-\frac{x}{m}} \right\}$$

that of C″ A″ B″ is evidently

$$y' = \frac{\frac{1}{3}\, y_0}{2} \left\{ E^{\frac{x}{m}} + E^{-\frac{x}{m}} \right\} \qquad (48.)$$

The principle of this example is general.

When the *extrados* is a transformed catenary, note that, since in all the formulæ under Case III., $w =$ the weight corresponding to a unit of surface of the space between C A B and M Y, we must make in these formulæ

$$w = n\, w'$$

Where $w' =$ weight of the building material and $n = \frac{\text{A A}''}{\text{Y A}}$.

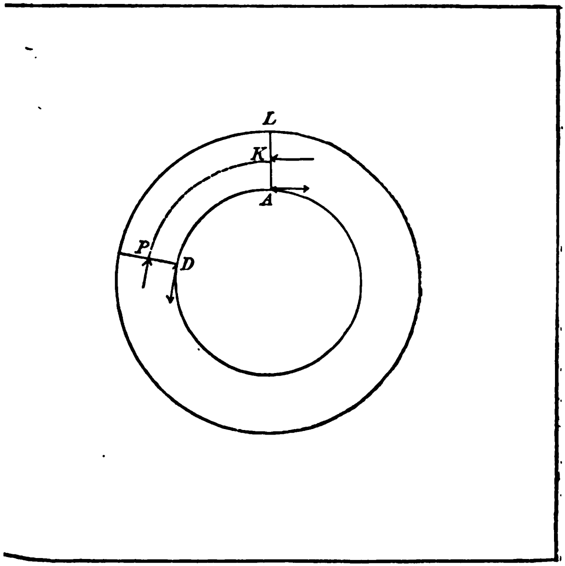
In arches of this class no provision is needed for horizontal thrust on the spandrels as the arch is equilibrated under vertical loads alone.

In all stone or brick arches, the changes in the curve of pressures K K′ due to passing loads are usually slight, because the weight of such passing loads is generally small compared with the weight of the arch itself and its backing.

3. The simplest practical case in which a uniform normal load (such as that discussed in Case IV.) can be applied to an arch is when it is subjected to water pressure, the arch ring being horizontal instead of ver-

tical. Such a pressure will exist on an empty well constructed in a reservoir or other body of water (Fig. 39). For each

FIG. 39.




horizontal layer of the well wall may be considered as subjected to a uniform normal pressure of an intensity due to the length of the water at that level. This intensity will of course diminish, and so will the pressure on the wall from one layer to another as we come towards the top.

The soffit of such a wall should be [illegible] from [illegible]. The thickness of the wall at any depth must be determined by the thrust, which is constant and equal at any given layer and is

$$T = [illegible] \quad . \quad . \quad [illegible]$$

Where $w$ = weight of a unit of water and $y$ = depth of water at the layer in question.

In determining the line of pressures consider a section of the wall between two vertical planes not parallel as heretofore, but both normal to the soffit. Take for the lever-arm of the couple at A (Fig 39) a distance A K = ½ A L. The *force* is still to be = H = T.

At D apply an equal couple with force = the thrust along the soffit at that point, which is also = T = H. Then the lever arm must be equal to A K. Hence we see that the curve of pressures is a circle *parallel* to the soffit and may pass through the middle of the arch ring.

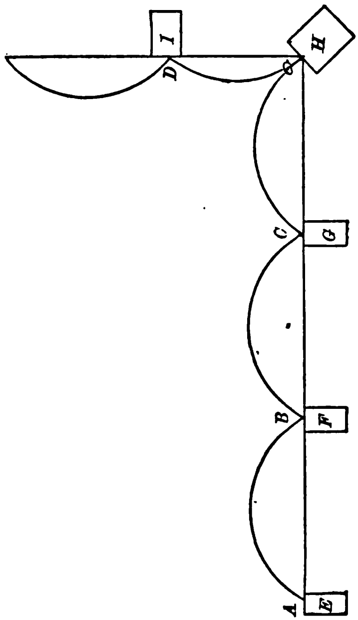


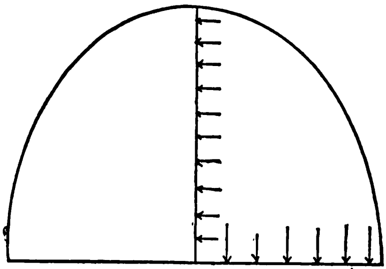

Fig. 40.

This kind of arch may be used for dams or the walls of reservoirs. (See Fig. 40.)

4. There is no case in ordinary practice

where the pressures upon an arch are strictly identical with those on an elliptical cord, for in this case, the pressure must be constant in intensity along both the horizontal and vertical projections of the arch, but the intensity along the *horizontal* must differ from that along the vertical in a constant ratio (Fig. 41). But, as Prof. Ran-

FIG. 41.



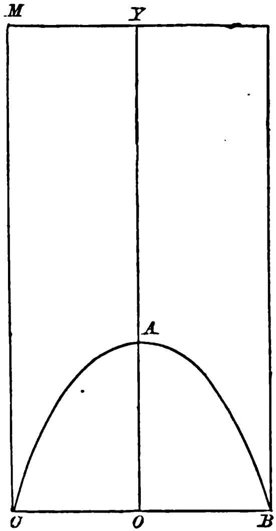
kine says, the curve of equilibrium for the arch of a tunnel through earth, when the depth below the surface is great compared with the rise of the arch itself, approximates to an ellipse.

The pressures in a mass of earth are intermediate in character between those exng in a solid and those in a liquid mass. as a little cube of earth (Fig. 42) under weight of the superincumbent column earth $p$, presses downward with a force

FIG. 42.


al to its own weight and that of the umn above. It also presses out horizon- y with a force *less than this d* ce, but always *bearing a con* *to it.* If the little cube were *solid* it would have *no* horizontal push; if *liquid*, that horizontal push would equal its pressure downward. If the upper surface of the earth is

FIG. 43.




inclined, the outward push which always remains parallel to it becomes inclined too, and is then "conjugate" to the vertical.

If M Y (Fig. 43) is the surface of the earth, when Y A is great compared with A O, then Y A and M C differ so slightly that we may assume them to be equal. We then have on the arch a uniform vertical load whose intensity =

$$p_y = (\text{Y A}) \times \text{weight of a unit of the earth} = wy_0;$$

and a horizontal load whose uniform intensity $p_x$ is equal to the vertical intensity ($p_y$) multiplied by a constant. Let

$$\frac{p_x}{p_y} = c^2 \text{ (a constant).}$$

Then

$$p_x = c^2 wy_0 \text{ and } c = \sqrt{\frac{p_x}{p_y}}.$$

From the discussion of Case V. we see that $c$ must be the ratio of the axes of the ellipse to which the pressures are respectively parallel. Hence if the arch be a semi-ellipse and O B be given, we have

$$\frac{\text{O B}}{\text{O A}} = c \;\therefore\; \text{O A} = \frac{\text{O B}}{c}.$$

From these data draw the curve of the soffit.

The thrust along the soffit at A

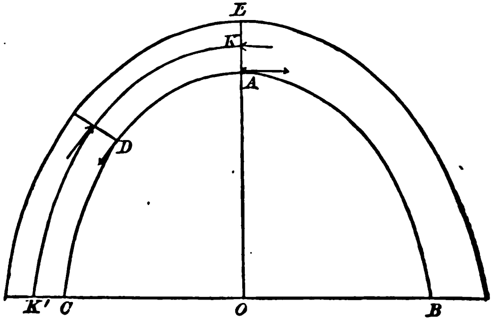
$$= \mathrm{H} = p_y \rho_0 = w y_0 \rho_0 .$$

At C or B it is $\mathrm{V} = p_x \rho_1$.

At other points it may be gotten from eq. (27) Case V.

We can determine the curve of pressures by a method similar to that used in the last case. Here, however, the curve K K′ will not be parallel to C A, since the thrusts along C A are not constant, but increase from A to C. Assume A K (Fig. 44)

FIG. 44.




$= \frac{2}{3}$ A L, then the arch must be so proportioned that K K′ shall fall within the mid-third.

If the arch C A B is not to be a *semi-ellipse* (as above assumed) but only a *segment* of one, a few trials will enable us to get the ellipse from the data already given.

The strictly true curve of equilibrium required by earth pressure is the Geostatic arch.

5. An arch built with the curve discussed in *Case VI.*, is known as the Hydrostatic arch, from the fact that the loading there described is similar to the pressure of water upon a *vertical* arch.

For if M Y (Fig. 45) be the surface of the water, then its pressure on C A B is normal and proportioned at each point to the depth below M Y. This pressure, as has been shown, may be resolved into a vertical and horizontal pressure at each point, this vertical and horizontal pressure being equal in *intensity* to each other at every point, and also to the normal pressure of which they are the components.

The above form of arch may be applied in two cases.

(1) To bear the pressure of water or other

liquid. Thus in the case of a river tunnel (such as those at Chicago) where the top of

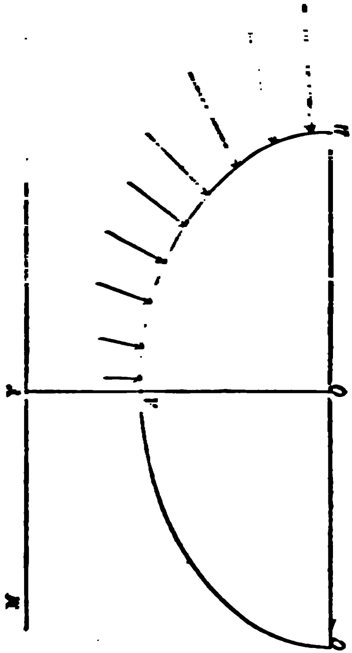


FIG. 43.

the tunnel is practically on a level with the bottom of the river, we might use the hydrostatic arch.

The equation of the curve is

$$y\rho = y_0 \rho_0.$$

The vertical load on the half-arch A B

$$= \int_{x_1}^{0} p dx = \mathrm{V} = w y_1 \rho_1 = \text{thrust along arch at B.}$$

The horizontal pressure against A B

$$= \int_{y_1}^{y_0} p dy = w \frac{y_1{}^2 - y_0{}^2}{2} = \mathrm{H} = w y_0 \rho_0. \quad (51)$$

The thrust along the arch is constant, or

$$\mathrm{T} = \mathrm{H} = \mathrm{V}.$$

The rise A O ($= a$), the depth A Y ($= y_0$), and the radii at A and B ($\rho_0$ and $\rho_1$) are connected by the following approximate equations. The co-ordinates of B being $x_1$ and $y_1$, let

$$b = x_1 + \frac{x_1{}^2}{30\,a}. \quad \text{Then } y_0 = a \frac{a^3}{b^3 - a^3} \quad (52)$$

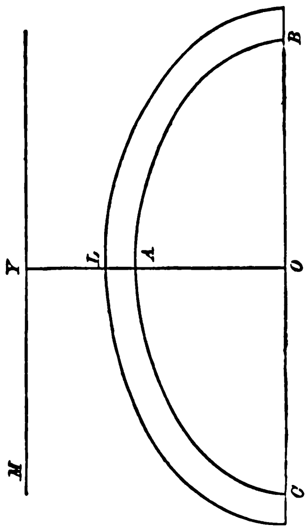
$$\rho_0 = \frac{y_1{}^2 - y_0{}^2}{2\,y_0} = a + \frac{a^2}{2\,y_0} = \frac{a}{2}\left(1 + \frac{b^3}{a^3}\right) \quad (53)$$

$$\rho_1 = \frac{y_1{}^2 - y_0{}^2}{2\,y_1} = a - \frac{a^2}{2(y_0 + a)} = \frac{a}{2}\left(1 + \frac{a^3}{b^3}\right). \quad (54)$$

The line of pressures in a hydrostatic arch, since T is constant, is *parallel* to the soffit, as in circular arches.

*Example.*—Suppose the span to be 50 ft. (Fig. 46) and the depth A Y = 16 ft.

Fig. 46.




Find first the rise A O. In Eq. (52) $x_1 = 25'$ $y_0 = 16'$, and a few trials show that $a$ = rise = 20′ about.

Hence

$$\rho_0 = 32\tfrac{1}{2} \text{ ft. and } \rho_1 = 14.1 \text{ ft.}$$

With these data describe the curve of the soffit—the radius at any other point besides A and B being given by the equation

$$\rho = \frac{y_0 \rho_0}{y}.$$

The thrust at $A = H = w y_0 \rho_0$. Here $w = 62.4$ lbs.

$$\therefore H = 32448 \text{ lbs.}$$

The rule for the depth of keystone in a single arch gives

Depth $AL = \sqrt{.12 \times 32.5} = 1.9$ ft.

This is ample. It only gives about 120 lbs. per sq. in. as the pressure at the crown.

T being $= H$, the depth of the arch-ring may be uniform.

(2) The hydrostatic arch is also used when the loading is homogeneous masonry up to the extrados M Y, *provided the spandrels be suited to sustain a horizontal thrust at each point of the arch equal to the vertical load at that point.*

As all stone or brick arches sink at the crown when the centres are removed, they will exert at other points an outward horizontal thrust. Now if we assume that this horizontal thrust is at every point equal in *intensity* to the vertical loading at that point, the curve of equilibrium under such a system of forces is the hydrostatic curve. This is the assumed condition of the forces acting in the Neuilly and other bridges of this class.

When the spandrels cannot be made firm and solid this form should not be used, but when they can be, as in the successive arches of a stone bridge, it is advantageous rather than otherwise, to have such a thrust from the arch against the spandrel; while the hydrostatic curve of given span and rise gives a greater *water-way* than the corresponding catenary would.

The catenary needs no resistance from the spandrel, being balanced under the vertical load alone.

*Example.*—Let the span be 100 ft. and rise 30 ft. Then the depth of loading at the crown (= A Y, Fig. 46) will be found from equation 52

$$= y_0 = 7\tfrac{1}{3} \text{ ft.}$$

Then $$\rho_0 = 91.7 \text{ ft.}$$

Hence $H = wy_0\rho_0$ (putting $w = 160$ lbs.) $= 107600$ lbs.

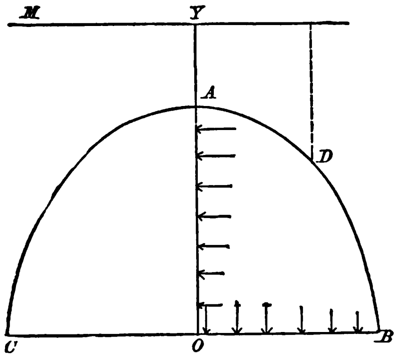
Depth of keystone

$$= \sqrt{.12 \times 91.7} = 3.3 \text{ ft.}$$

This gives a pressure of 32,280 lbs. to the sq. ft., or about 225 lbs. to the sq. in.

6. If the vertical forces vary as in the hydrostatic arch, and the *horizontal* are not equal to them, but differ at each point in a constant ratio, the curve of equilibrium

FIG. 47.




(Fig. 47) becomes the Geostatic curve discussed in *Case VII.* This curve derives its name from the fact that the system of pressures above described is similar to that exerted by a mass of loose earth against C A B. Let M Y = the horizontal surface of the earth: then at each point D of the arch there is a vertical pressure of intensity ($p'_y$), proportional to the depth ($y$) of D below M Y, and a horizontal pressure whose intensity is less than $p'_y$ in a constant ratio,

or $$p'_x = c^2 p'_y$$

($c^2$ being taken to represent the ratio of the intensities).

Assume a hydrostatic arch whose vertical dimensions shall be identical with those of the geostatic arch, and whose span (C B) (Fig. 48) shall be connected with the span of the geostatic arch (C′ B′) by the equation

$$C B = \frac{C' B'}{c}. \qquad (55)$$

The intensity of the vertical pressure (the horizontal is like it) in this hydrostatic arch must be

$$p_y = c p'_y .$$

From these data deduce a hydrostatic arch, and then pass by parallel projections to the required geostatic arch.

Equations (35) (36) (37) (38) give the values of the quantities needed in discussing the Geostatic arch.

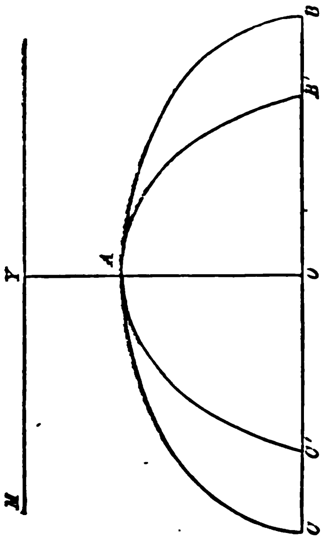



Fig. 48.

*Example* 1.—Let the span of the geostatic arch (C′ B′ = 100 ft.) be given; also the depth of the loading (A Y = 20 ft.);

also the ratio of the pressures ($c^2 = \frac{1}{3}$); and the weight of a cubic ft. of the loading $= w = 100$ lbs. Whence

$$p'_{y_0} = wy_0 = 2000 \text{ lbs.}$$

Then since

$$\text{C B} = \frac{\text{C}'\,\text{B}'}{c} = \frac{100}{\sqrt{\frac{1}{3}}} = 172.4 \text{ ft.}$$

$$cw = 58 \text{ lbs.}$$

$$py_0 = cp'y_0 = \sqrt{\tfrac{1}{3}}\,.2000 = 1154.7 \text{ lbs.}$$

We find from equations (52) (53) (54) for the hydrostatic arch

$$\text{Rise} = a = \text{O A} = 57.7 \text{ ft.}$$

$$\rho_0 = 140.93 \text{ ft.}$$

$$\rho_1 = 36.3 \text{ ft.}$$

$\text{H} = \text{V} = \text{T} = py_0\,\rho_0 = 1154.7 \times 140.93 = 162.700$ lbs. nearly.

In the geostatic arch we have from equations (35 (36) (37) and (38)

Thrust at $\text{B} = \text{V}' = \text{V} = 162700$ lbs.

" $\text{A} = \text{H}' = c\,\text{H} = 94300$ lbs. nearly.

$\rho_0' = 46.97$ ft. $\rho_1' = 62.65$ ft.

*Example* 2.—Suppose the span $= 100$ ft. depth, $\text{A Y} = y_0 = 20$ ft. and rise, $a = 30$ ft. given; to find $c$ and thence the hydrostatic arch.

From equation (52) we find

$$b = 40.71,$$

and thence in same equations $x_1 = 39$. Hence the span of the hydrostatic arch

$$= 2x_1 = 78 \text{ ft.}$$

And as $c.CB = C' B'$

$$c = \frac{100}{78} = 1.28.$$

Then proceed as in the last example. In this example the hydrostatic arch is the smaller of the two.

The line of pressures in a geostatic arch is found as it was in the elliptic.

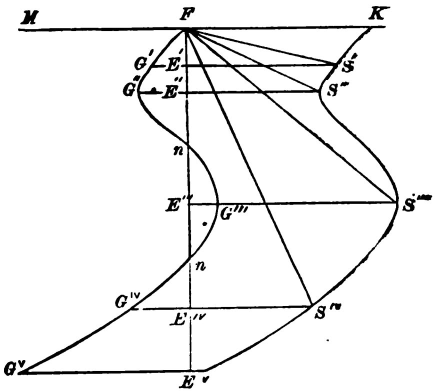
The geostatic is the true curve of equilibrium under earth pressure, but when A Y (Fig. 48) is great compared with A O, it approximates the ellipse described through the points C′ A B′ as already stated.

7. Convenience, or other reasons, will often dictate the form of the arch without reference to the loading, and again, necessity may make the vertical load different from any and all the cases we have discussed. In such instances Case VIII. will

enable us to determine the character and amount of the horizontal forces which must be applied through the resistance of the spandrel, when once the form of the arch and the vertical load are known.

When the horizontal forces thus required are *thrusts* directed against the arch, it is

FIG. 50.




generally possible so to build the spandrel that the arch may be secure, but when they are the opposite, or *outward pulls* on the arch, then it is difficult to insure stability, as to do so requires tension between the arch and the spandrel. In such cases it is best to change the form of the arch.

The discussion of Case VIII. of cords, enables us to determine the necessary data in the case of similar linear arches under similar loads.

Fig. (50) gives the geometrical construction of the triangle of forces at every point of the semi-arch A B (Fig. 49).

We may discuss a given linear arch C A B under a given vertical load, by determining:

1. Thrust at crown; which is

$$H_0 = p_0 \rho_0. \quad (56)$$

2. Total horizontal thrust required on any arc A D′, A D″, etc. This, from equation (42), is

$$H = H_0 - V \cot. i. \quad (57)$$

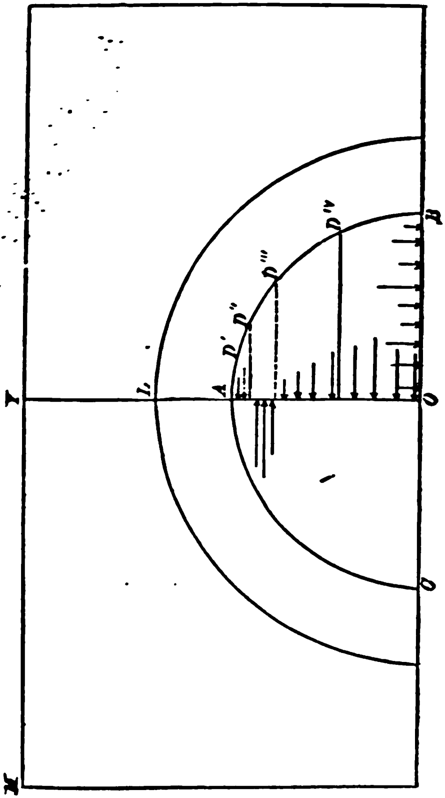
If this be negative the spandrel must exert a *pull* instead of a thrust.

On the half-arch A B the above equation becomes

$$H_1 = H_0 - V_1 \cot. i_1. \quad (58)$$

On any arc B D$^{v}$, counting from B upwards, the total spandrel thrust is

FIG. 49



$$H_1 - H = - V_1 \cot. i_1 + V \cot. i. \qquad (59)$$

This last expression has at least one maximum value corresponding to some arc B D. In the Fig. (49) this value corresponds to the arc B D‴.

Let this maximum value be denoted by $H_m$ and let $i_m$ = the inclination at D‴. Then

$$H_m = - V_1 \cot. i_1 + V \cot. i_m = E''' G'''. \qquad \text{(Fig. 50)} \quad (60)$$

D‴ is known as the "point of rupture." There the action of the spandrel ceases to be a thrust, and must, above that point, for some distance at least, become tension.

3. The *intensity* of the horizontal spandrel thrust or pull in any layer (as between D‴ and $D^{v}$) is from equation (43)

$$p_x = - \frac{d H}{dy} = - \frac{d (V \cot. i)}{dy} = - \frac{d\left(V \frac{dx}{dy}\right)}{dy}.$$

When H is positive (that is thrust) $p_x$ is negative, as it should be, since it is equal to the *increment* of the abscissas of the curve F G′ G″, etc. (Fig. 50), and these increments are decreasing from G‴ to $G^{v}$.

At the point of rupture

$$p_x = 0. \qquad (61)$$

We can determine the point of rupture in three ways: First, by constructing the Fig. (50) and finding the inclination ($i_m$) corresponding to the maximum abscissa E‴ G‴. Secondly, by substituting the various values of $i$ and V in the value of

$$(H_1 - H) \text{ (eq. 59)},$$

and getting the maximum value of the expression. The $i$ which gives this maximum value corresponds to the point of rupture. Thirdly, by solving equation (61) $p_x = 0$.

4. The thrust along the rib at every point is from equation 40,

$$T = V \text{ cosec. } i, \qquad (62)$$

and it is represented by the inclined lines F S′ F S″, etc., Fig. (50).

The *horizontal component* of this thrust is

$$H_r = V \text{ cot. } i = \text{the abscissas of K S}', \text{ etc.},$$

which are always equal to $H_0$, the thrust at the crown *minus* the spandrel thrust between A and the point in question

$$\therefore H_r = V \text{ cot. } i = H_0 - H. \qquad (63)$$

This is evidently a maximum at the point of rupture, or, since at the point of rupture,

$$H = H_1 - H_m,$$

we have

$$H_R = H_0 - H_1 + H_m.$$

But

$$H_0 - H_1 = V_1 \cot. i,$$

$$\therefore \quad H_R = V_1 \cot. i_1 + H_m. \qquad (64)$$

This horizontal thrust of the rib at D‴ is therefore to be balanced by the horizontal reaction of the abutment at B ($= V_1 \cot. i_1$) togther with the resistance of the spandrel between B and D‴ ($= H_m$). When the arch is vertical at B, $V_1 \cot. i_1 = 0$.

5. In single arches it is necessary to know the point of application of the resultant of the forces represented by ($V_1 \cot. i_1 + H_m$) in order to determine the stability of the abutments. Take moments with reference to the axis of abscissas M Y. Then if $y_R =$ ordinate of point in question, and $y_m$ and $y_1$ be the ordinates of D‴ and B, we have

$$H_R y_R = (V_1 \cot. i_1) y_1 + \int y dH$$

$$= (V_1 \cot. i_1) y_1 + \int_{y_m}^{y_0} y p_x \, dy.$$

$$y_2 = \frac{[illegible] - \int_{y_m}^{[illegible]} [illegible]\, dy}{H_2}. \qquad (65)$$

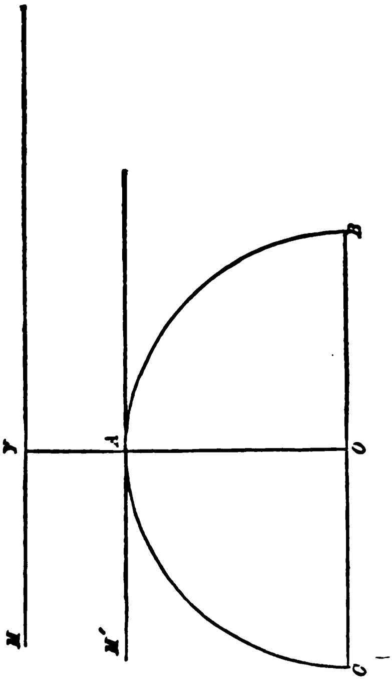



Fig. 51.

In this we neglect the spandrel forces above

D''' so far as they affect the stability of the abutment. This can be done with safety.

The line of pressures and depth of keystone are determined as heretofore.

*Example* 1.—Let the assumed form of the soffit be a semi-circle, and let the loading consist of the arch and backing of homogeneous masonry carried up to a horizontal "extrados" M Y (Fig. 51).

Place the radius of the arch $= r$

Depth A Y $= a\,r$

Heaviness of the material $= w$

Take the origin of co-ordinates at A and express the co-ordinates in terms of the inclination $i$ of the arch as on p. 217 *Rankine's C. E.*

Then

Thrust at crown $= \mathrm{H}_0 = p_0\,\rho_0 = (war)\,v = war$

Vertical load on any arc $= \mathrm{V} = wr^2$

$$\left\{(a+1)\sin.\ i - \frac{\cos.\ i \sin.\ i}{2} - \frac{i}{2}\right\}$$

Spandrel thrust on any arc A D

$$\mathrm{H} = \mathrm{H}_0 - \mathrm{V}\cot.\ i = wr^2$$

$$\left\{a - (1+a)\cos.\ i + \frac{\cos.^2 i}{2} + \frac{i\cos.\ i}{2\sin.\ i}\right\}$$

On A B this becomes (since the arch is vertical at B and C)

$$H_1 = war^2 = H_0$$
$$\therefore H_m = V \cot. i_m.$$

Intensity of spandrel thrust

$$p_x = - \frac{d\,(V \cot. i)}{dy} = wr$$
$$\left\{ (1 + a) - \cos. i - \frac{i - \cos. i \sin. i}{2 \sin^2 i} \right\}$$

The point of rupture is found by putting $p_x = o$ and finding the value of $i_m$ by trials. As a first approximation

$$i_m = \text{arc. cos. } \frac{1 + 3a}{2}$$

Thrust along the rib $= T = V \operatorname{cosec} i$. At B this is

$$V_1 = wr^2 \left(a + 1 - \frac{\pi}{4}\right).$$

So

$$H_R = V_1 \cot. i_1 + H_m = H_m = wr^2$$
$$\left\{ (1 + a) \cos. i_m - \frac{\cos.^2 i_m}{2} - \frac{i_m \cot. i_m}{2} \right\},$$

and

$$y_R = \frac{r^2}{H_R} \int_{i_m}^{90^\circ} p_x \sin. i\,(1 - \cos. i)\, di.$$

*Example* 2.—Let

$$r = 20' \quad A\,Y = 2\ 5'.$$

Then

$$a = \frac{2.5}{20} = \frac{1}{8} w = 150 \text{ lbs.} \quad \text{(Fig. 52.)}$$

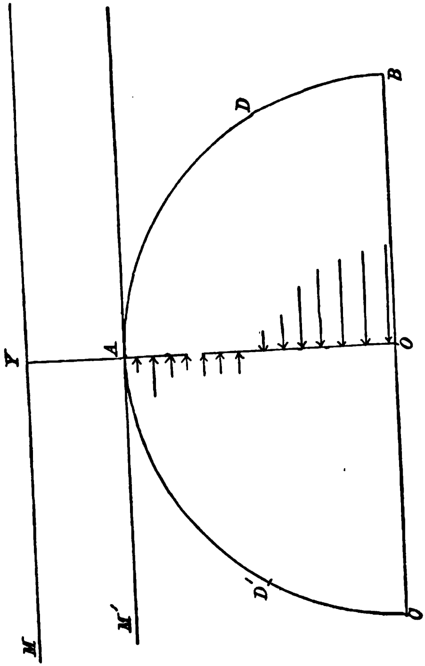



FIG. 52.

Then

$$\mathbf{H}_0 = war^2 = 7500 \text{ lbs.}$$

$$V = 60000 \left\{ \frac{9}{8} \sin. i - \frac{\cos. i \sin. i}{2} - \frac{i}{2} \right\}.$$

At B. $V = 60000 \left\{ \frac{9}{8} - \frac{\pi}{4} \right\} = 20376$ lbs.

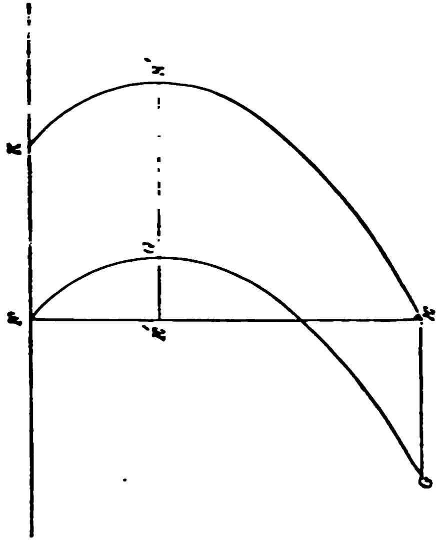


FIG. 53.

**Angle of rupture**

$$i_m = \text{arc. cos.} \frac{1 + \frac{3}{8}}{2} = \cos.^{-1} .6875 = 46° \, 34'$$

$$H_B = H_m = 60000$$

$$\left\{ \frac{9}{8} (.6875) - \frac{(.6875^2)}{2} - \frac{.81 \times .947}{2} \right\} = 8154 \text{ lbs.}$$

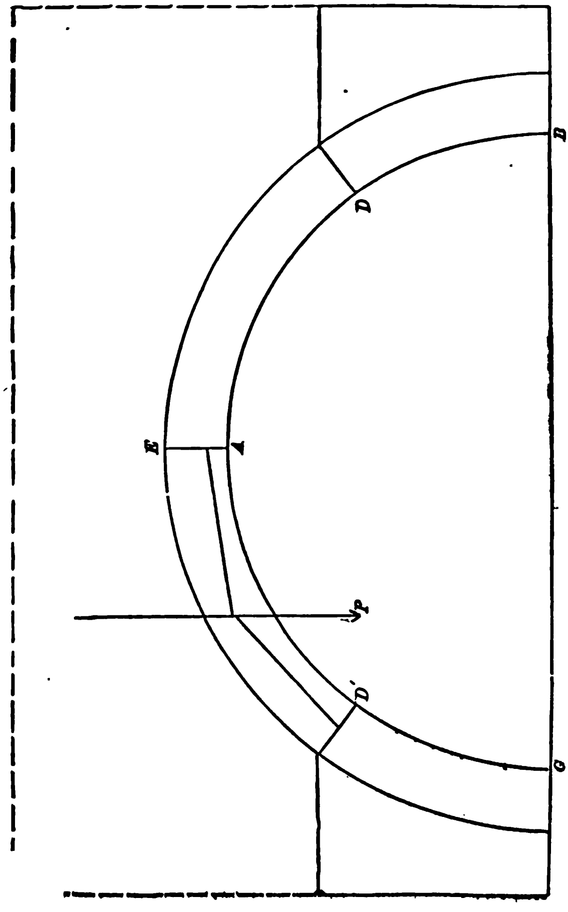
(Fig. 53) shows the manner in which the forces vary. From A to D (Fig. 52) there must be a pull in the spandrel to produce equilibrium. The total amount of this pull is small, being

$$= 8154 - 7500 = 654 \text{ lbs.}$$

To rid the arch of it, so that the part $D^1$ A D shall either be balanced under the vertical load alone or exert a thrust outwards, instead of a pull inwards, we *flatten* the arc $D^1$ A D. A few trials will determine this flattening near enough for practice.

Thus if $D^1$ A D (Fig. 54) is to be balanced under the vertical load alone, find the centre of gravity of the section $D^1$ A and its load. Draw a vertical line P through this point, then if we can draw a line from any point in the middle third of the joint $D^1$ parallel to the tangent to the arch there, and from its intersection with P draw a line parallel to the arch at A which will intersect A L within the middle third, then the extreme points of the line of pressures in the section A $D^1$ will be within the middle

Fig. 54.



third, and the line of pressures will generally be altogether within it.

The new radius required for the arc $D^1$ A D may also be determined roughly by putting $H_{,,} = war^2 = H_R$ and thence getting $r^1$ since, if $D^1$ A D is to be balanced under vertical load alone, the horizontal thrust at every point of it must be the same and $= H_R$, the thrust at $D^1$ and D.

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